Math Powerpacs A-F

Teacher's Guide

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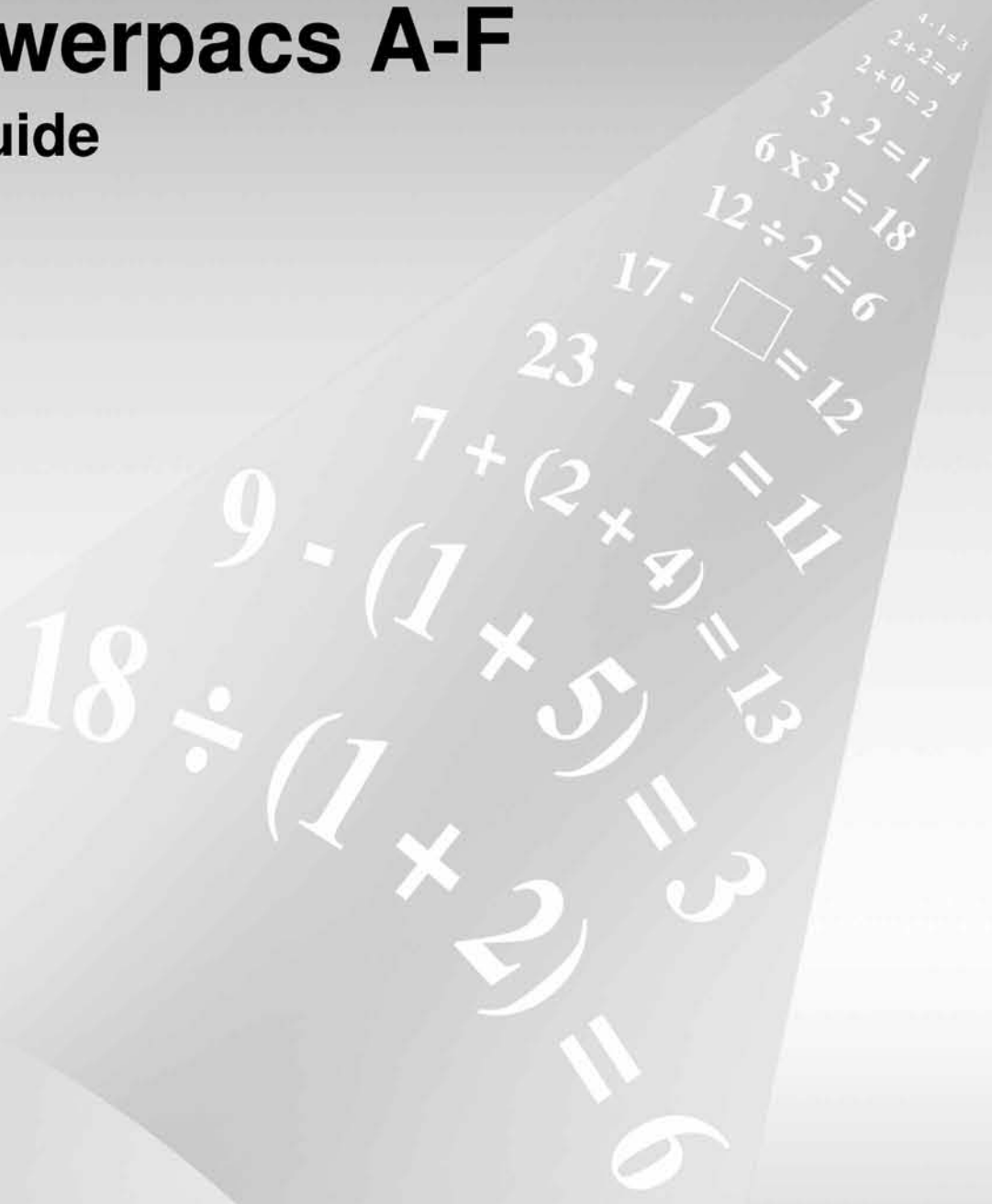

FORMULA I

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Formula 1 Math Powerpacs

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INTRODUCTION

TO THE TEACHER

Each of the six lessons per *Powerpac* represents a highly motivating approach to a basic math skill. Each concept is introduced in simple terms and is taught in a step-by-step sequence of increasing difficulty to build understanding and ensure success. The tone of each lesson is informal and positive, and a listen-respond-evaluate technique is used throughout to gain a high degree of student involvement.

This *Formula I Math Powerpac* is one of six which are available. Each *Powerpac* is designed to be a self-contained unit of instruction. A list of the other *Formula I Math Powerpacs* is provided on the last page of this manual for your convenience.

Use: Lessons in the *Powerpac* are self-directing and enable the student to work independently. However, they may also be used successfully in small group situations involving students with similar needs and abilities.

Organization: Each *audio,* with its accompanying student *activity pages,* constitutes a self-contained lesson. The lessons in the *Powerpac* may be used in any order, depending upon the needs and abilities of the individual student.

The Audio: Each *audio* begins with an informal greeting from the narrator, setting the tone for personal interaction which is maintained throughout the lesson. The narrator not only explains the material and directs the student's work, but contributes the strong personal support and encouragement so important in remedial work.

Occasional pauses are timed into the tape to allow the student to respond to the narrator's questions. For most of the actual problem solving, however, the student is directed to turn off the player and complete his work on the *activity page.* When he has finished the work prescribed by the narrator, he starts the player again and checks his answers at the narrator's direction.

The use of headsets is recommended for individual and small group use to avoid disturbing other students and to eliminate room noises.

The Student Response Pages: Each *audio* is accompanied by 4 *activity* reproducible student response pages. In most lessons, the first three pages are completed by the student as he listens to the *audio.* The responses on these pages are self-checked with answers given by the narrator. However, it is recommended that you look over the student's work at the end of the lesson to determine his degree of understanding of the concepts presented. For most of the lessons, the last page contains review exercises which the student is asked to complete after the *audio* has finished playing. His work on this page will allow you to ascertain the extent to which he has understood and applied the concepts presented in the lesson.

It should be noted that the narrator does not ask the student to mark incorrect responses. If you wish the student to do so, instruct him accordingly before the *audio* is started.

The Teacher's Manual: This teacher's manual contains individual lesson guides for each lesson in the *Powerpac.* The first part of each lesson guide includes an estimate of the time needed by the student to complete the entire lesson. It should be remembered that individual differences among students will account for some variation in this time.

Under the section entitled *Background Needed,* you will find a statement of the prior knowledge a student should have to derive the optimum benefit from the lesson. The section entitled *In the Lesson,* contains a summary of skills and topics presented in the order in which they are introduced on the *audio.* Also provided in this section are special instructions concerning the organization of the *activity pages* or the need for scratch paper or a ruler.

Under the *Vocabulary* heading, the special terms and symbols introduced in the lesson are defined. The *Evaluation* portion deals with the review page of the *activity pages,* usually page 4, which is completed by the student after the *audio* has finished playing.

For convenience in evaluating student performance, each lesson guide includes reproductions of the response *activity pages* with answers overprinted.

Formula 1 Math Powerpac A

LESSON 1

Sets

Approximate time required to complete the lesson: 25 min.

Background Needed

Odd and even numbers are used by the narrator in many of the examples of sets which are presented on the ***audio.*** The student must therefore have a working knowledge of these two sets of numbers in order to complete this lesson successfully.

In The Lesson

Following an introduction to set notation, the student learns to distinguish between finite and infinite sets and works with the empty set. A diagram on page 2 of the response booklet provides the basis for a number of problems involving intersection and union of sets. Finally, the ideas of equivalent, equal, and disjoint sets are presented and the student is asked to differentiate between them.

Vocabulary: These terms and symbols are introduced on the ***audio.***

set: a collection of objects or numbers, indicated by braces; i.e., {1, 2, 3, 5}

finite set: a set in which all members can be listed

infinite set: a set in which all members cannot be listed; indicated by three or four members and a series of dots; i.e., {0, 2, 4, ...}

members: the items in a set

equal sets: sets with identical members

equivalent sets: sets which contain the same number of members

nonequivalent sets: sets which do not have the same number of members

empty set: a set with no members

disjoint sets: sets which have no members in common

union: a joining of the members of sets without repeating the common members; shown by ∪

intersection: the set of members common to two or more sets; shown by ∩

"n" (followed by name of set in parentheses): indicates the number of members in a set; e.g., n(D) = 3, means there are three members in Set D or the number property of the Set D is 3

1. A = {1, 3, 5, 7, 9}

2. B = {0, 2, 4, 6}
   Set B is made up of the first 4 even numbers.

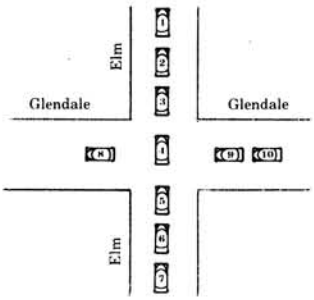
3. G = {t, c, a}
   Is set G finite? yes

4. W = {0, 1, 2, 3, ...}
   Can you write the largest whole number? no

5. C = {1, 2, 3, ...}
   D = {e, j, t}
   Which set is finite? D
   Which set is infinite? C

6. Z = { }

7. A = { }
   Set A has 0 members.
   B = {0}
   Set B has 1 member.
   Which set is empty? A



8. E = {1, 2, 3, 4, 5, 6, 7}

9. G = {8, 4, 9, 10}

10. E ∪ G = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

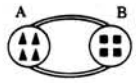
11. E ∩ G = {4}

12. J = {c, a, t}
    K = {g, r, a, y}
    J ∪ K = {c, a, t, g, r, y}
    J ∩ K = {a}

13. C = {1, 3, 5}
    D = {2, 4, 6}
    C ∪ D = {1, 3, 5, 2, 4, 6}
    C ∩ D = { }
    C ∩ D is called the empty set.

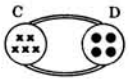
14. D = {d, o, g}
    F = {f, i, s, h}
    D ∪ F = {d, o, g, f, i, s, h}
    D ∩ F = { }
    n(D) = 3
    n(F) = 4
    n(D ∪ F) = 7
    n(D ∩ F) = 0

2

15. 

n(A) = 4 n(B) = 4

16. Equivalent sets are sets which have the same number of members. Sets A and B are equivalent.

17. 

n(C) = 5 n(D) = 4

Are sets C and D equivalent? no

18. A = {1, 2, 3, 4}
B = {1, 3, 5, 7}
Are sets A and B equivalent? yes
A ∩ B = {1, 3}
A ∪ B = {1, 2, 3, 4, 5, 7}

19. C = {1, 3, 5, 7}
D = {2, 4, 6}
Do sets C and D have any members in common? no
Sets with no members in common are disjoint sets.

20. E ∩ F = { }
Sets E and F are disjoint sets.

21. B = {c, a, t}
E = {t, a, c}
Do sets B and E have the same members? yes
When two sets have the same members, they are called equal sets.

22. Are equal sets equivalent sets? yes

23. Are all equivalent sets equal sets? no

3

1. A = { }
A is the empty set.

2. E = {0, 2, 4, ...}
T = {0, 5, 10}
Which set is finite? T
Which set is infinite? E

3. Is the set of even numbers finite or infinite? infinite

4. Is the set of odd numbers finite or infinite? infinite

5. B = {1, 2, 3}
1, 2, 3 are called the members of set B.

6. A = {2, 3, 6}
B = {1, 2, 3, 4, 5}
A ∪ B = {1, 2, 3, 4, 5, 6}
A ∩ B = {2, 3}

7. A = {2, 4, 6} B = {1, 3, 5}
C = {5, 2, 3} D = {6, 4, 2}
Which sets are equal? A & D
Sets A, B, C, and D are equivalent sets.

8. C ∩ D = { }
C and D are disjoint sets.

9. E = {g, o, a, t}
F = {b, i, r, d}
E ∪ F = {g, o, a, t, b, i, r, d}
E ∩ F = { }
n(E) = 4 n(F) = 4
n(E ∪ F) = 8 n(E ∩ F) = 0

10. E = {0, 2, 4, ...}
Set E is the set of even numbers.
Set E is an infinite set.

11. Equivalent sets have the same number of members.

12. A set with no members is called the empty set.

4

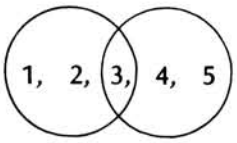
Evaluation: The student's performance on page 4, which he completes after the ***audio*** has finished playing, should give you an indication of his understanding of the concept of sets.

A Step Further

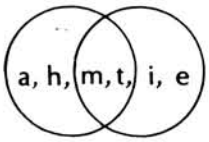
The following activities will provide your more able students an opportunity to further explore topics related to sets.

1. A Venn diagram, consisting of two or more shapes, usually circles, can be used effectively to demonstrate intersection of sets. The members of each set are enclosed within a circle, and the common members are displayed in the region of intersection of the circles diagrammed below. Put the following examples on the board as a guide and have students diagram the intersections of two problems of their own creation.

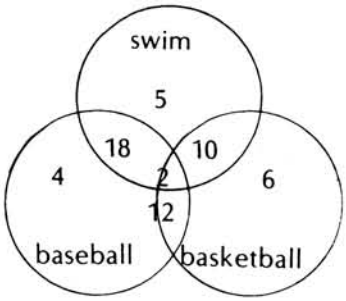
   Examples: A = {1, 2, 3}
   B = {3, 4, 5}
   A ∩ B = 3



D = {m, a, t, h}
E = {t, i, m, e}
D ∩ E = m, t



2. Introduce students to the use of a three-circle Venn diagram which provides a clear picture of how sets are related. Subsets and intersection of sets are illustrated in the example below in which the three circles are used to show the distribution of students participating in three different sports activities. There are 36 students who play baseball, 35 students who swim, and 30 students who play basketball. Eighteen of these students swim and play baseball, 10 swim and play basketball, and 12 play baseball and basketball, while 2 students participate in all three sports.



LESSON 2

Addition Facts through 20

Approximate time required to complete the lesson: 28 min.

In The Lesson

Following introduction of the terms *addend* and *sum*, the narrator introduces the number line as a tool for solving addition facts. The student hears explanations of the commutative property, the identity element for addition, and the associative property and practices using them in solving problems.

Special Instructions: Page 3 of the booklet is completed by the student immediately after the *audio* is finished. He is told by the narrator to complete page 4, which provides practice of a more challenging nature, only if the teacher has asked him to do so. You must tell the student, before the *audio* begins, whether or not he is to complete the fourth page.

Vocabulary: The following terms are introduced and used on the *audio.*

addends: the numbers that are added in an addition problem

sum: the result or answer in addition

commutative property of addition: the property which allows the order of addends to be changed without affecting the sum

associative property of addition: the property which allows the addends to be grouped in any way without affecting the sum

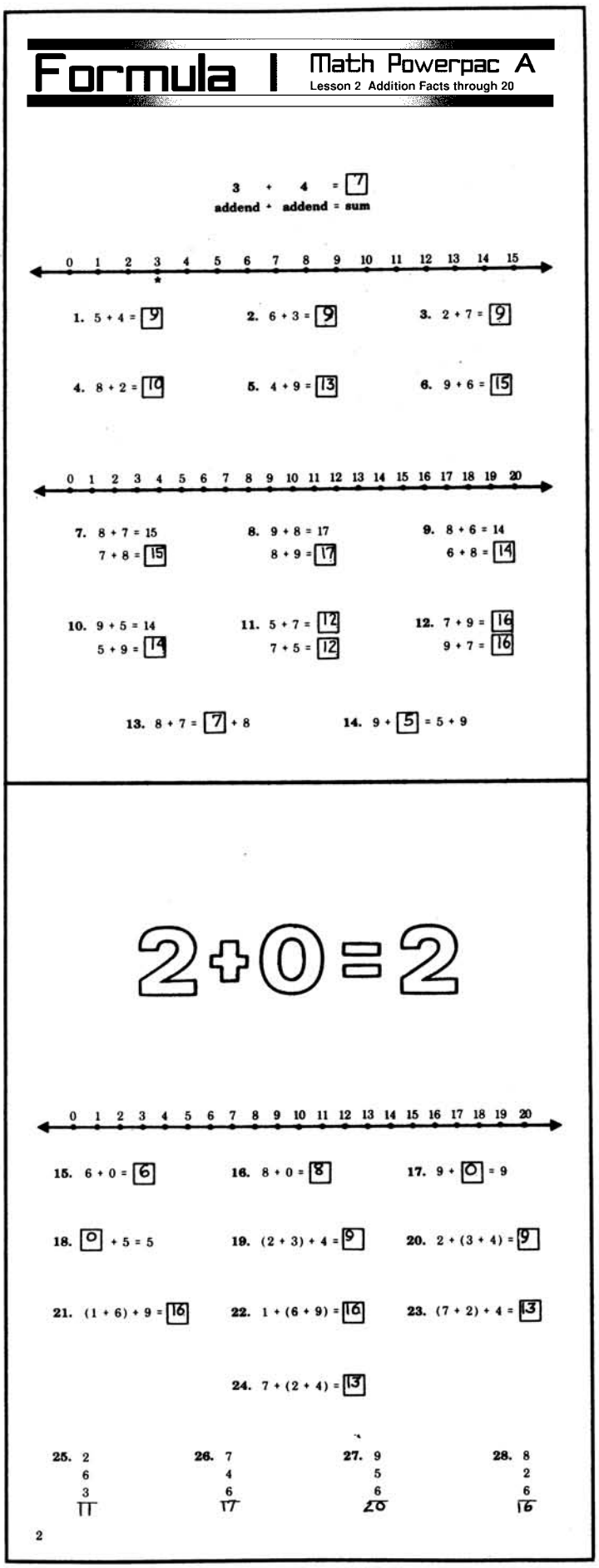
identity element for addition: the number which, when added to any number, is that number; zero is the identity element of addition

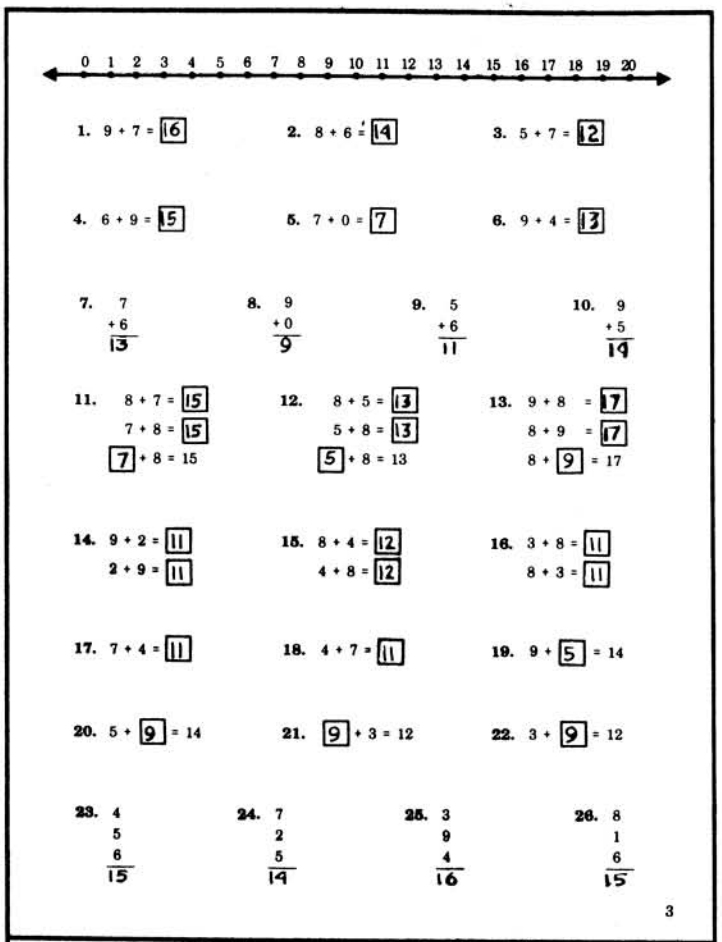
Evaluation: The student's performance on page 3 of the response booklet, which he completes after the *audio* has finished playing, should give you an indication of his mastery of addition facts through 20 and his understanding of the properties of addition.

A Step Further

If students have had no difficulty with the work in the response booklet, it is suggested that you use the following activities to help them further explore addition facts through 20.

1. Play "Number Scrabble" by following these simple steps:





0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

1. 9 + 7 = 16
2. 8 + 6 = 14
3. 5 + 7 = 12
4. 6 + 9 = 15
5. 7 + 0 = 7
6. 9 + 4 = 13

7.	8.	9.	10.
7	9	5	9
+6	+0	+6	+5
13	9	11	14

11. 8 + 7 = 15; 7 + 8 = 15; 7 + 8 = 15
12. 8 + 5 = 13; 5 + 8 = 13; 5 + 8 = 13
13. 9 + 8 = 17; 8 + 9 = 17; 8 + 9 = 17
14. 9 + 2 = 11; 2 + 9 = 11
15. 8 + 4 = 12; 4 + 8 = 12
16. 3 + 8 = 11; 8 + 3 = 11
17. 7 + 4 = 11
18. 4 + 7 = 11
19. 9 + 5 = 14
20. 5 + 9 = 14
21. 9 + 3 = 12
22. 3 + 9 = 12

23.	24.	25.	26.
4	7	3	8
5	2	9	1
6	5	4	6
15	14	16	15

3

Work carefully to find the answer to each addition problem below.

1. 5 + (4 + 9) = 5 + 13 = 18
2. (6 + 1) + (8 + 5) = 7 + 13 = 20
3. 12 + 1 + 7 + 0 = 13 + 7 = 20
4. 5 + 4 + 4 + 5 + 2 = 20
5. (5 + 4) + 9 = 9 + 9 = 18
6. (2 + 13) + (2 + 2) = 15 + 4 = 19
7. 9 + 3 + 2 + 5 = 12 + 7 = 19
8. 3 + 2 + 1 + 3 + 11 = 20

Read each problem carefully before you compute the answer.

9. In a football game against Eastside, the Westside team scored 7 points in the first quarter, 3 points in the second quarter, 6 points in the third quarter, and 3 points in the last quarter.

The Eastside team scored 2 points in the first quarter, 12 points in the second quarter, no points in the third quarter, and 6 points in the last quarter. Who won the game and by how many points?

Eastside by 1 point

Work space:

Westside	Eastside
7	2
3	12
6	0
3	6
19 points	20 points

10. Don and Bill each have a paper route. They wanted to increase the number of customers on their routes. Don was able to get 2 customers on each weekday, and 5 customers on Saturday. Bill got 3 new customers on each of the first two days, 1 customer on each of the next three days, and 7 customers on Saturday. Who got more customers, and how many more customers did he get than the other boy?

Bill got 1 more customer than Don

Work space:

Don	Bill
2	3
2	3
2	1
2	1
2	1
5	7
15 customers	16 customers

4

a. Rule a 9- x 12-inch sheet of construction paper into 1-inch squares (or use graph paper with 1-inch squares).
b. Cut 70 1-inch squares (tiles) from tagboard.
c. On 50 of the squares, write bold numerals, using five squares for each digit from zero through 9.
d. On the remaining 20 tiles, write large plus signs.
e. Write a small numeral in the lower right-hand corner of each digit tile, using the following point system:

1 point for each 0, 1, 2, and 3 square
2 points for each 4, 5, and 6 square
3 points for each 7, 8, and 9 square

Rules: Place all squares face down and have each player (four may play) select ten of the squares, which he places in his view, but hidden from the view of his opponents. The starting player places three tiles (two digits separated by a plus sign) on the board either vertically or horizontally. He then draws, from the "extras" pile, the number of tiles he has used. The next player follows in turn, placing two of his tiles on the board. He can play horizontally or vertically, but not diagonally. His tiles must be adjacent to one of the tiles (either digit or a plus sign) already on the board. A plus sign must separate any two digits on the board. No points are counted until a player completes a sequence of tiles which add up to a sum designated by the teacher for that game. For example, if the teacher has designated that game as "sum 14," the only time a player earns a point is when the tiles he has played complete a sequence of addends which totals 14. The player's score is counted by adding the small corner numbers of the addends which constitute "sum 14."

		6_2			
	6_2	+	5_2	+	3_1
	+	8_3			
	3_1				

If a player is unable to play at least two of his tiles, he loses a turn. However, he may return the unplayable tiles to the face-down pile and select a corresponding number of new ones. The game continues until the pile is gone and one player has exhausted his supply of tiles. At this point, the player with the most points is declared the winner.

LESSON 3

Subtraction Facts through 20

Approximate time required to complete the lesson: 32 min.

Background Needed

Throughout this lesson, addition facts are used to teach the meaning of subtraction. Therefore, to benefit fully from the lesson, the student should know his addition facts through 20.

In The Lesson

The operation of subtraction is introduced as the inverse of addition. The terms *addend* and *sum* are used extensively in the lesson as the listener is guided to look for the missing addend in subtraction problems, often using the number line as an aid. The student learns that a missing addend problem can be stated as either an addition sentence or a subtraction sentence. He also discovers that any number subtracted from itself is zero and that subtracting zero from a number does not change that number.

Vocabulary: The following term is introduced and used in this lesson.

inverse: opposite; subtraction is the *inverse* of addition; often called additive *inverse*

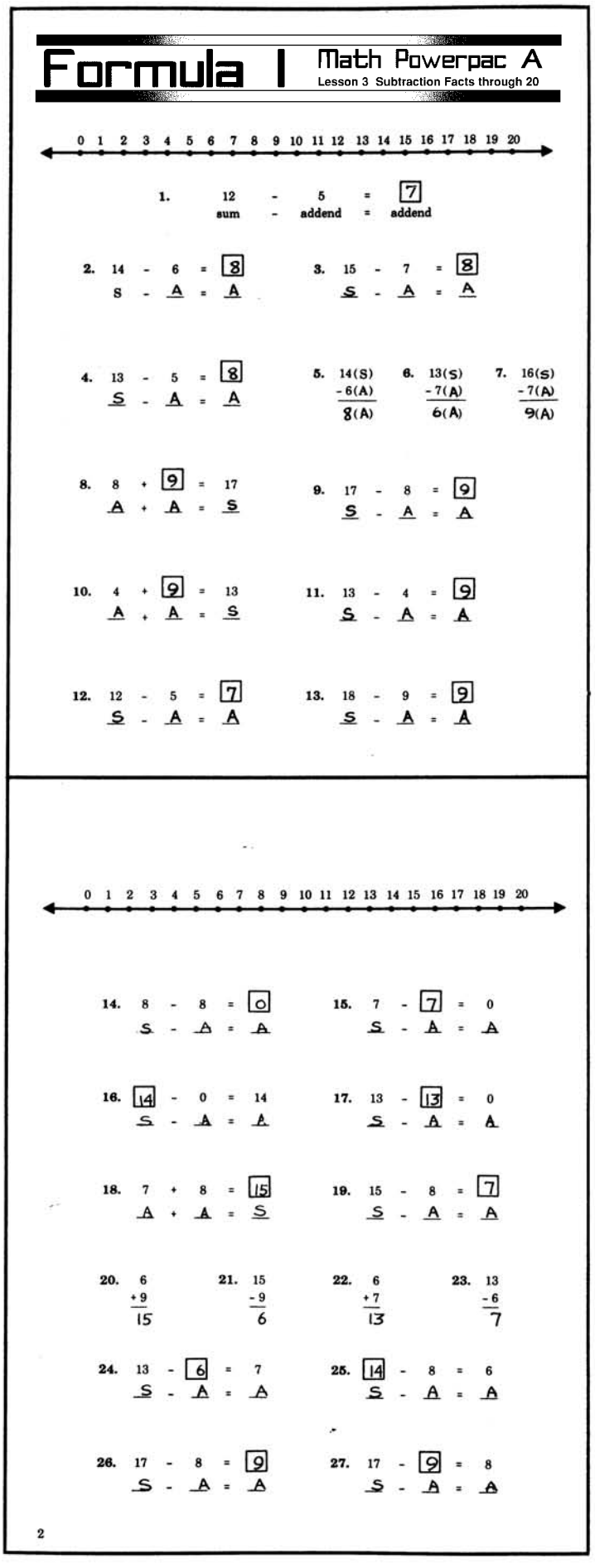
Evaluation: The student completes page 4 of the response booklet after the *audio* has finished playing. His performance on this page should give you an indication of how well he has mastered the subtraction skills taught in this lesson.

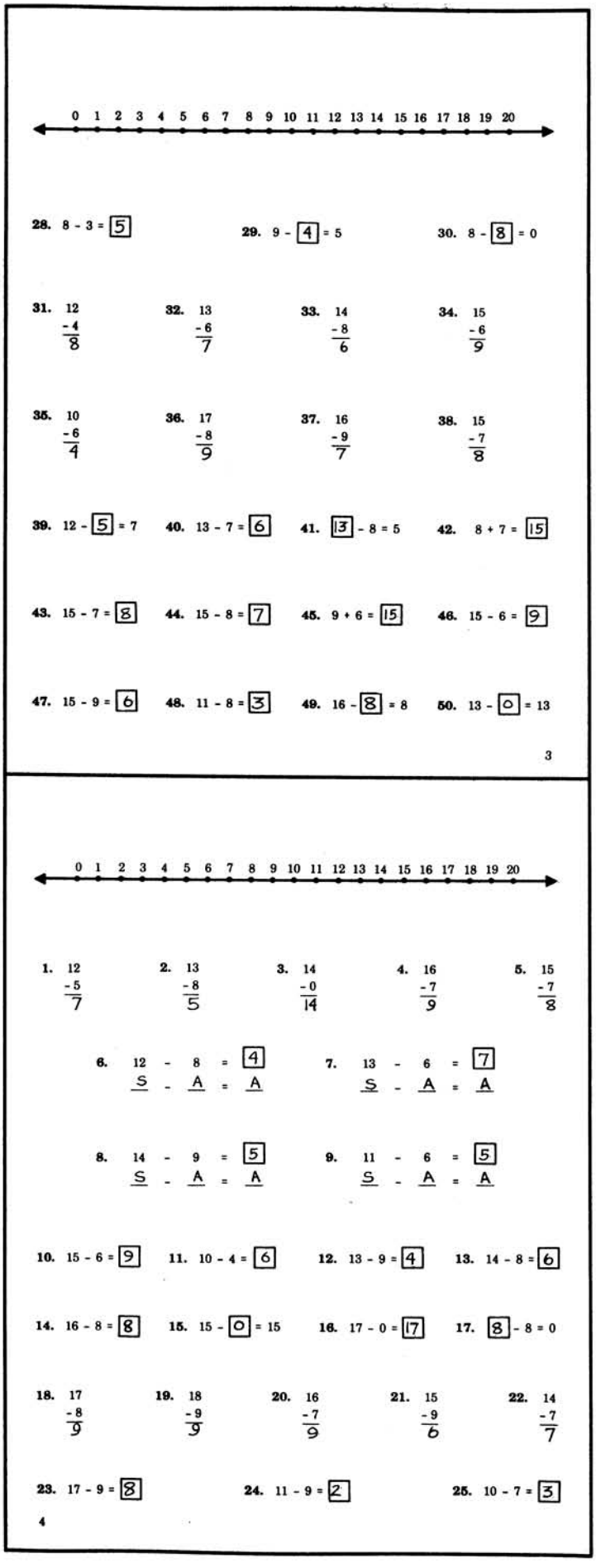
A Step Further

If students have had no trouble with the work in the response booklet, it is suggested that you use the following activities to help them further explore subtraction.

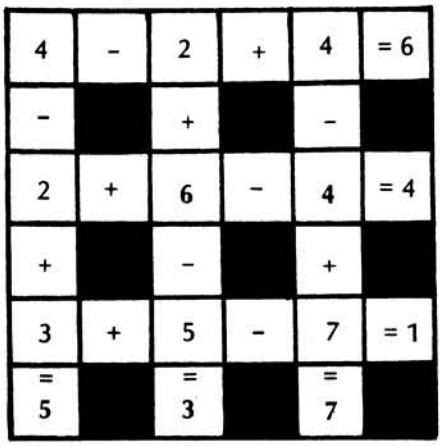
1. "Crossnumber" puzzles can provide challenging practice in the use of addition and subtraction. Display the puzzle shown below, leaving out the red numerals and signs, and have students fill in the squares so that the number sentences will be correct both horizontally and vertically.

   If time permits, you might wish to construct several different puzzles for your students. To do so, start in the upper left square and construct all the problems for the puzzle. When the puzzle is complete, simply take out the symbols and num-



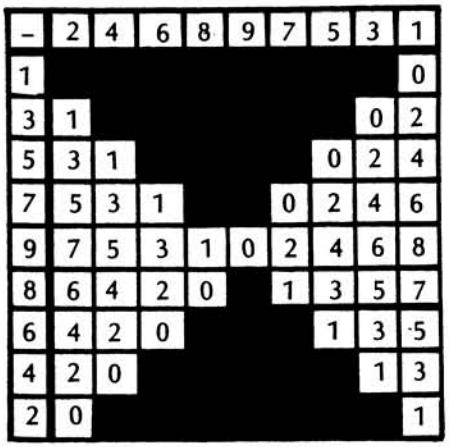


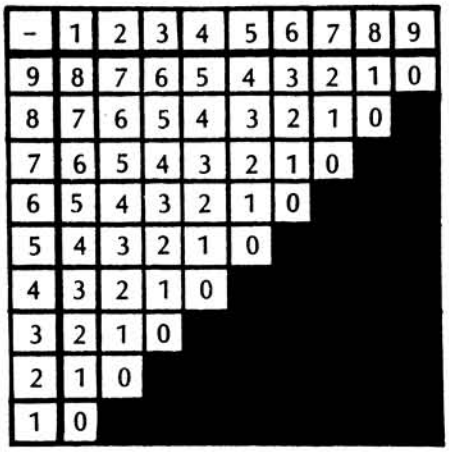
bers which you wish the student to "discover." More advanced students might construct "cross-number" puzzles for each other.



3. In the number puzzles below, the numbers in the top horizontal row are subtracted from the numbers in the vertical row at the left, and the answers are written in a square below the subtrahend. When the subtraction results in a negative number, the square is shaded. Have students experiment by rearranging the addends and sums to make several different patterns with the shaded squares.

Here are examples of patterns that may result from changing the order of the sums and addends.





LESSON 4

Multiplication Facts through 45

Approximate time required to complete the lesson: 22 min.

In The Lesson

Simple illustrations of sets, arrays, and number lines are used in this lesson to increase the student's understanding of multiplication. The narrator guides the student to recognize and use the commutative property, the property of zero, and the identity element for multiplication.

Vocabulary: The following terms are introduced and used on the *audio.*

factors: the numbers that are multiplied

product: the answer in a multiplication problem

rectangular array: an orderly visual representation of multiplication; a rectangular pattern of dots in which the number of rows represents one factor and the number of dots in each row, the other factor; the total number of dots in the array represents the product

commutative property of multiplication: the property which allows the order of factors to be changed without changing the product

property of zero in multiplication: when zero is a factor, the product is always zero

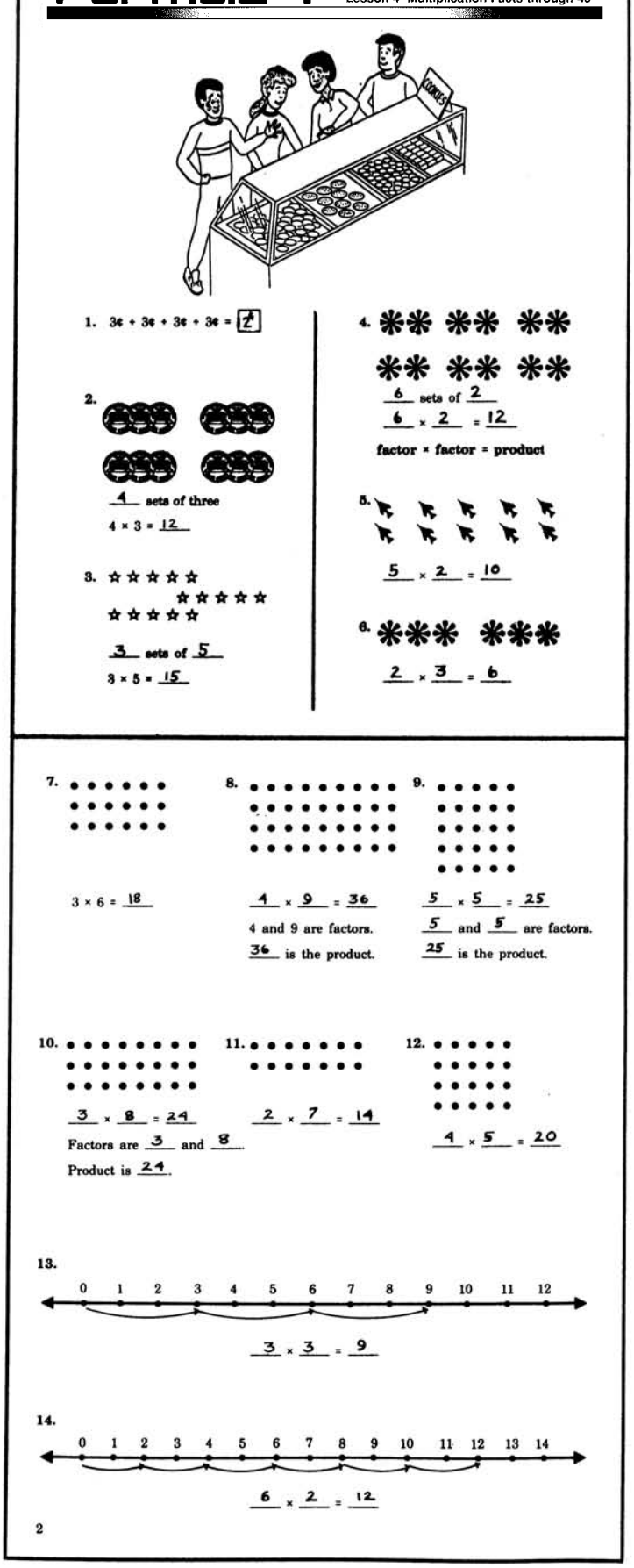
identity element for multiplication: when one is a factor, the product is always the same as the other factor

Evaluation: The student's performance on page 4 of the response booklet, which he completes after the *audio* has finished playing, will give you an indication of his ability to use multiplication facts through 45.

A Step Further

If students have had no difficulty with the work on the response pages, you may wish to use the following activities to help them explore in greater depth multiplication facts through 45.

1. Display the Egyptian numeration system at the top of page 9; then have students complete the following problems by inserting the correct signs.



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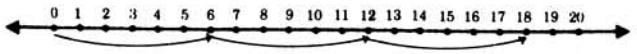
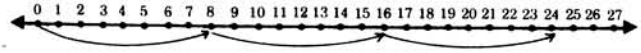


3 × 6 = 18

16.



3 × 8 = 24

17. 3 × 5 = 15
5 × 3 = 15

18. 5 × 9 = 45
9 × 5 = 45

19. 4 × 6 = 24
6 × 4 = 24

20. 7 × 2 = 14
2 × 7 = 14

21. 4 × 0 = 0

22. 3 × 0 = 0

23. 0 × 7 = 0

24. 9 × 0 = 0

25. 8 × 0 = 0

26. 0 × 7 = 0

27. 5 × 1 = 5

28. 4 × 1 = 4

29. 6 × 1 = 6

30. 1 × 3 = 3

31. $\begin{array}{r} 7 \\ \times 3 \\ \hline 21 \end{array}$

32. $\begin{array}{r} 3 \\ \times 7 \\ \hline 21 \end{array}$

33. $\begin{array}{r} 8 \\ \times 4 \\ \hline 32 \end{array}$

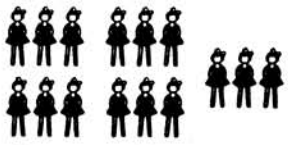
34. $\begin{array}{r} 4 \\ \times 8 \\ \hline 32 \end{array}$

3

1.

3 × 4 = 12

2.



5 × 3 = 15

3. $\begin{array}{r} 7 \\ \times 2 \\ \hline 14 \end{array}$

4. $\begin{array}{r} 0 \\ \times 6 \\ \hline 0 \end{array}$

5. $\begin{array}{r} 6 \\ \times 4 \\ \hline 24 \end{array}$

6. $\begin{array}{r} 8 \\ \times 1 \\ \hline 8 \end{array}$

7. 3 × 6 = 18

8. 4 × 7 = 28

9. 3 × 9 = 27

10. In 3 × 7 = 21 3 and 7 are factors.

11. 2 × 8 = 16
8 × 2 = 16

12. 4 × 9 = 36
9 × 4 = 36

13. 5 × 7 = 35
7 × 5 = 35

14. $\begin{array}{r} 8 \\ \times 3 \\ \hline 24 \end{array}$

15. $\begin{array}{r} 9 \\ \times 0 \\ \hline 0 \end{array}$

16. $\begin{array}{r} 6 \\ \times 5 \\ \hline 30 \end{array}$

17. $\begin{array}{r} 5 \\ \times 9 \\ \hline 45 \end{array}$

18. $\begin{array}{r} 7 \\ \times 1 \\ \hline 7 \end{array}$

19. 4 × 8 = 32

20. 6 × 3 = 18

21. 5 + 5 + 5 = 3 × 5 or 15

22. 2 × 6 = 12
3 × 6 = 18

23. 2 × 5 = 10
4 × 5 = 20

24. 3 × 4 = 12
6 × 4 = 24

4

Numeration System:

1 = I 2 = II 3 = III 4 = IIII 5 = IIIII

6 = IIIIII 7 = IIIIIII 8 = IIIIIIII 9 = IIIIIIIII 10 = ∩

100 = C 1,000 = 

Problems:

a. IIIIIIIIIIIIIIIII c. IIIIIIIIIIII

b. ∩ ∩I ∩∩I d. ∩ IIIIIIIIIIII ∩∩∩∩∩∩II

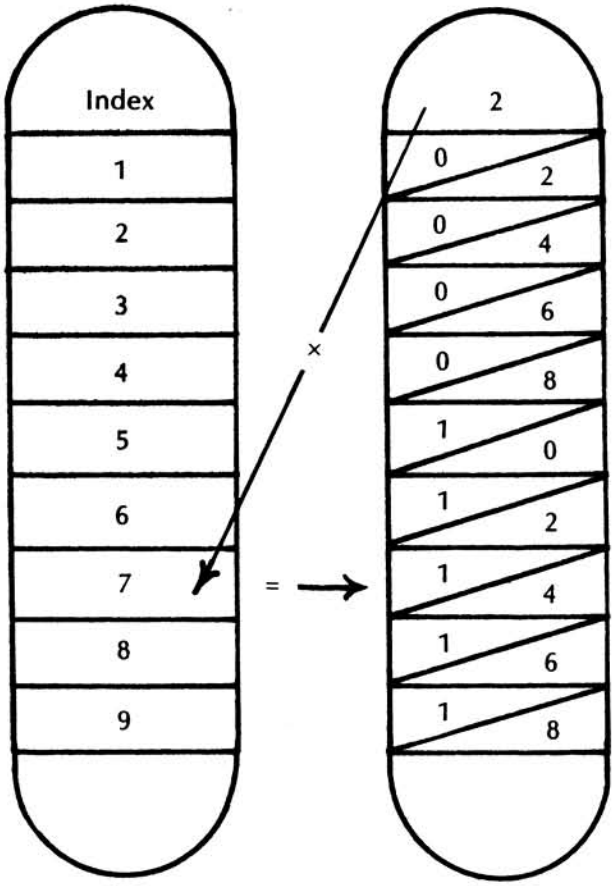
Answers:

a. × = c. = × or ÷ =

b. + = d. × + =

If time permits, you might expand the activity by composing more difficult problems or by having students make up problems for each other.

2. For some interesting practice in multiplication, construct a set of Napier's Rods showing the multiplication facts of 2, 3, 4, and 5. Either tongue depressors or strips of cardboard may by used. The index and factor rods should be set up as in the examples below. The diagonal lines separate tens and ones. To multiply with the rods, simply place the index rod next to the desired factor, as shown, and read the result of any desired multiplication fact on the factor rod. The example below shows 2 × 7 = 14.



LESSON 5

Multiplication Facts through 81

Approximate time required to complete the lesson: 35 min.

Background Needed

Since this lesson is based upon the first multiplication facts, it is important that the student should have mastered the multiplication facts through the product of 45.

In The Lesson

The narrator guides the student in using the distributive property of multiplication over addition by combining smaller multiplication facts to find larger ones. He also reviews the commutative property, the property of zero, the identity element for multiplication, and the associative property. An interesting characteristic of 9 is introduced: when 9 is a factor, the digits in the product add up to nine.

Special Instructions: The student should be equipped with a sheet of scratch paper. Page 3 is the post *audio* evaluation page for this lesson. Page 4 consists of enrichment exercises, and the student is instructed to complete the page only if his teacher has told him to do so. It is important that you tell the student, before the *audio* begins, whether or not he is to complete the fourth page.

Vocabulary: The following term is introduced and used on the tape.

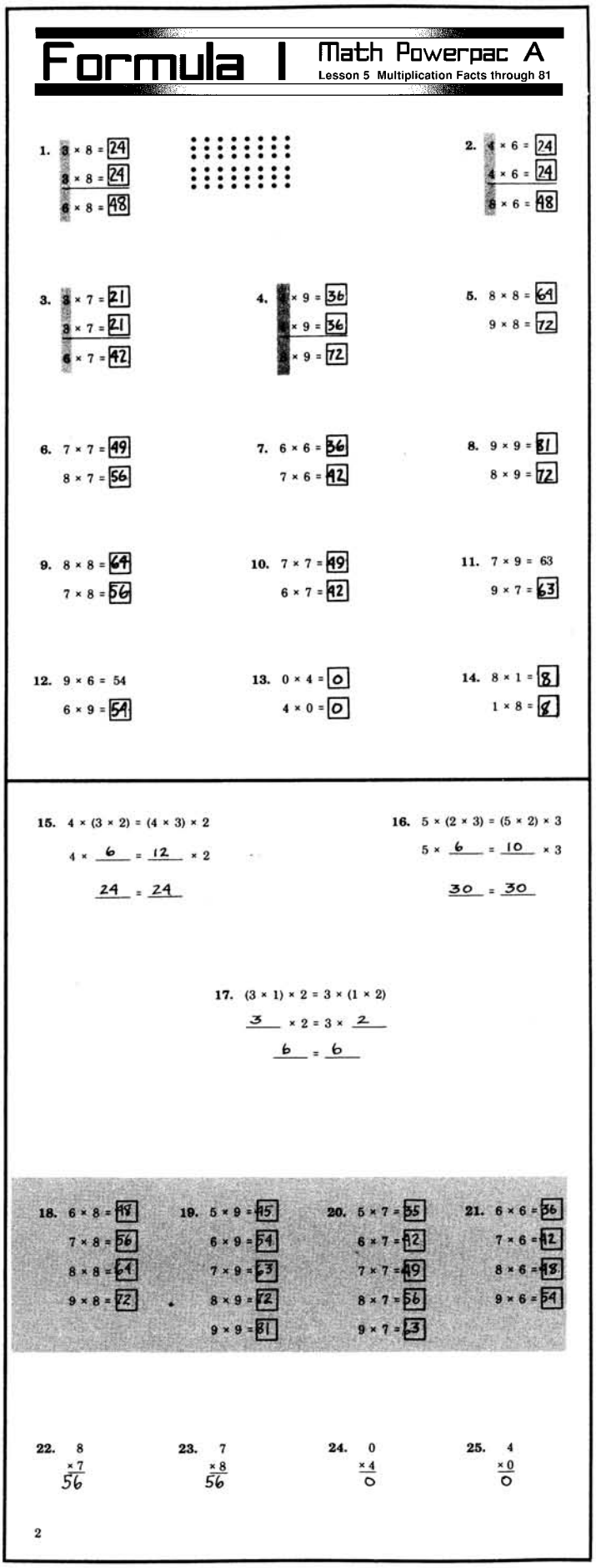
associative property of multiplication: the property which allows for variable grouping of three or more factors without changing the product

Evaluation: The student's performance on page 3 of the response booklet, which he completes after the *audio* has finished playing, will give you an indication of his mastery of the more difficult facts.

A Step Further

The following activities are included to provide able students with an opportunity to expand their knowledge of the concepts presented in this lesson.

1. Prepare a series of challenging exercises by giving the student problems which will require him to fill in the necessary factors, as shown



1. $3 \times 8 = 24$ $6 \times 8 = 48$
2. $4 \times 7 = 28$ $8 \times 7 = 56$
3. $4 \times 9 = 36$ $8 \times 9 = 72$
4. $3 \times 9 = 27$ $4 \times 9 = 36$ $7 \times 9 = 63$
5. $8 \times 0 = 0$
6. $5 \times 7 = 35$
7. $9 \times 1 = 9$
8. $8 \times 6 = 48$
9. $7 \times 9 = 63$
10. $5 \times 6 = 30$
11. $9 \times 6 = 54$
12. $7 \times 8 = 56$
13. $7 \times 8 = 56$ $8 \times 7 = 56$
14. $8 \times 9 = 72$ $9 \times 8 = 72$
15. $6 \times 9 = 54$ $9 \times 6 = 54$
16. $6 \times 6 = 36$ $6 \times 7 = 42$
17. $7 \times 7 = 49$ $7 \times 8 = 56$
18. $8 \times 8 = 64$ $8 \times 9 = 72$
19. $8 \times 5 = 40$
20. $6 \times 7 = 42$
21. $9 \times 8 = 72$
22. $8 \times 7 = 56$
23. $9 \times 7 = 63$
24. $(2 \times 3) \times 4 = 2 \times (3 \times 4)$
$6 \times 4 = 2 \times 12$
$24 = 24$
25. $(2 \times 4) \times 5 = 2 \times (4 \times 5)$
$8 \times 5 = 2 \times 20$
$40 = 40$

3

1. $7 \times \boxed{9} = 63$
2. $\boxed{8} \times 6 = 48$
3. $6 \times \boxed{7} = 42$
4. $\boxed{6} \times 9 = 54$
5. $7 \times \boxed{8} = 56$
6. $\boxed{9} \times 8 = 72$
7. $\boxed{8} \times 8 = 64$
8. $9 \times \boxed{0} = 0$
9. $\boxed{1} \times 7 = 7$

Each empty square in the charts below can be filled in with a number that is either a product, a sum, an addend, or a factor. For instance, in problem 10 in column a, $7 + 5 = 12$ and 7×5 is 35. Fill in the rest of the empty squares.

10.

	a	b	c	d	e	f
Sum	12	14	13	17	15	13
	7	6	6	9	9	5
	5	8	7	8	6	8
Product	35	48	42	72	54	40

11.

	a	b	c	d	e	f
Sum	12	15	17	13	15	14
	6	7	9	6	9	6
	6	8	8	7	6	8
Product	36	56	72	42	54	48

4

in the following examples:

$\square \times \square = 56$ $\square \times \square = 28$
$\square \times \square = 64$ $\square \times \square = 20$
$\square \times \square = 27$ $\square \times \square = 81$

This exercise may be made more difficult by using greater products.

2. Give students the "magic multiplication square" shown below and ask them to complete the following steps:
   a. Put a circle around any number in the square.
   b. Cross out all the numbers that appear in the same horizontal row and vertical column as the number selected.
   c. Repeat the process, selecting three other numbers (not already crossed out).
   d. Circle the one number that is left.
   e. Multiply the five circled numbers and keep the answer a secret.

When the student has found the answer, surprise him by telling him what his answer is. It will be 120,960 no matter which five numbers were circled.

8	6	10	14	2
4	3	5	7	1
32	24	40	56	8
24	18	30	42	6
12	9	15	21	3

Explanation: The square is a multiplication table in disguise. The second horizontal row contains one set of factors and the other set is contained in the fifth column. The common factor, one, is written in the square where the second row and fifth column intersect. The product of each two factors (one from the second row, the other from the fifth column) is written in the square where the row and column of the chosen factors intersect. Using a table structured in this manner, the product of any five numbers, selected at random, will always equal the product of all the factors.

You can construct other "magic squares" with other sets of factors in different rows and columns by remembering that the number where the factor row and factor column intersect must be one.

LESSON 6

Division Facts through 81

Approximate time required to complete the lesson: 45 min.

Background Needed

In this lesson, division facts are used interchangeably with multiplication facts. It is therefore essential that the student be able to multiply through the product 81 before starting with the ***audio.***

In The Lesson

The narrator presents division as the inverse operation of multiplication, sometimes referring to division problems as "missing factor" multiplication problems. The lesson begins with problems involving sets printed in the booklet and then progresses to more abstract exercises. Throughout the presentation, multiplication is used as a check for division problems. Both the horizontal and long forms of division are included.

Vocabulary: These terms are introduced and used on the ***audio.***

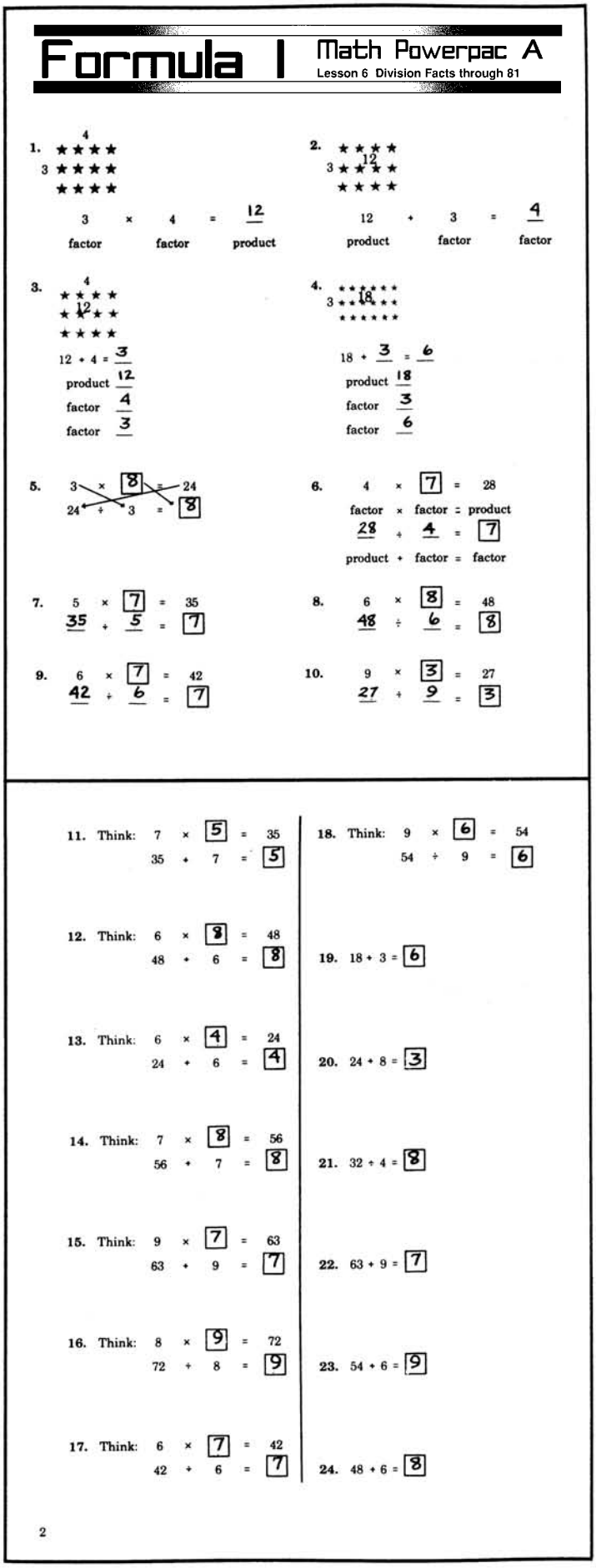
quotient: the answer in a division problem
dividend: the number to be divided
divisor: the number the dividend is divided by

Evaluation: The student's performance on page 4 of the ***activity page,*** which he completes after the ***audio*** has finished playing, should give you an indication of his ability to use the concepts presented in this lesson.

A Step Further

The following activities are suggested to give students an opportunity to further explore concepts related to division.

1. Ask students to join the numbers with the proper signs in the "operations puzzle" below. Depending upon the students' abilities, you might include only the signs or, perhaps, the



25. Think: $3 \times ? = 15$

$\boxed{5}$ quotient

$3\overline{)15}$ divisor) dividend

26. Think: $5 \times ? = 45$

$\boxed{9}$

$5\overline{)45}$

27. Think: $7 \times ? = 42$

$\boxed{6}$

$7\overline{)42}$

28. Think: $6 \times ? = 54$

$\boxed{9}$

$6\overline{)54}$

29. $\boxed{4}$

$7\overline{)28}$

Check: $7 \times 4 = 28$

30. $\boxed{4}$

$8\overline{)32}$

Check: $\underline{8} \times \underline{4} = \underline{32}$

31. $\boxed{8}$

$7\overline{)56}$

Check: $\underline{7} \times \underline{8} = \underline{56}$

32. $\boxed{9}$

$8\overline{)72}$

Check: $\underline{8} \times \underline{9} = \underline{72}$

33. $\boxed{7}$

$9\overline{)63}$

Check: $\underline{9} \times \underline{7} = \underline{63}$

34. $\boxed{8}$

$6\overline{)48}$

Check: $\underline{6} \times \underline{8} = \underline{48}$

35. $\boxed{8}$

$9\overline{)72}$

Check: $\underline{9} \times \underline{8} = \underline{72}$

36. $\boxed{7}$

$8\overline{)56}$

Check: $\underline{8} \times \underline{7} = \underline{56}$

3

1. $7 \times \boxed{8} = 56$

$56 \div 7 = \boxed{8}$

2. $\boxed{8} \times 9 = 72$

$72 \div 9 = \boxed{8}$

3. $6 \times \boxed{7} = 42$

$42 \div 6 = \boxed{7}$

4. $35 \div 7 = \boxed{5}$

$7 \times \boxed{5} = 35$

5. $56 \div 8 = \boxed{7}$

$8 \times \boxed{7} = 56$

6. $64 \div 8 = \boxed{8}$

$8 \times \boxed{8} = 64$

7. $72 \div 9 = \boxed{8}$

$9 \times \boxed{8} = 72$

8. $7\overline{)\overset{6}{42}}$

Check: $7 \times \underline{6} = 42$

9. $9\overline{)\overset{6}{54}}$

Check: $9 \times \underline{6} = 54$

10. $7\overline{)\overset{9}{63}}$

11. $6\overline{)\overset{9}{54}}$

12. $8\overline{)\overset{7}{56}}$

13. $49 \div 7 = \boxed{7}$

14. $81 \div 9 = \boxed{9}$

15. $48 \div 6 = \boxed{8}$

16. $35 \div 5 = \boxed{7}$

17. $21 \div 7 = \boxed{3}$

18. $24 \div 8 = \boxed{3}$

19. $63 \div 9 = \boxed{7}$

4

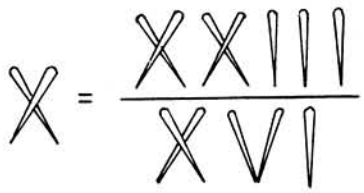
signs along with the numbers immediately following the equal signs.

(4	÷	2)	−	1	=	1
÷		×		+		×
(4	×	5)	÷	2	=	10
×		÷		−		÷
(4	−	2)	÷	2	=	1
=		=		=		=
(4	+	5)	+	1	=	10

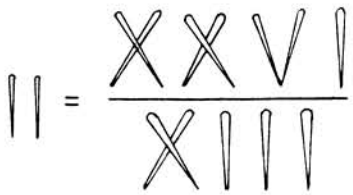
If students have no trouble with the first puzzle, you might give them the more difficult version below:

(81	÷	9)	−	6	=	3
−		×		÷		×
(9	×	4)	÷	6	=	6
÷		÷		+		−
(8	÷	4)	+	7	=	9
=		=		=		=
(9	+	9)	+	8	=	9

2. To play "Roman toothpicks," have students use toothpicks to set up the incorrect Roman numeral division problem shown below. Then, tell them to correct the problem by rearranging no more than six toothpicks. Toothpicks may not be transferred across the fraction bar or from one side of the equal sign to the other.



Answer:



Formula 1 Math Powerpac B

LESSON 1

Place Value through Millions

Approximate time required to complete the lesson: 30 min.

In The Lesson

A place value chart is used to illustrate the value of each digit within a numeral in base ten, and expanded notation is introduced to show the meaning of each digit. The narrator guides the student in learning place value through 100,000 and makes use of a period chart to teach the reading and understanding of greater numbers. All student responses on the first three pages of the booklet are immediately checked.

Vocabulary: The following terms are introduced and used in this lesson.

- **base 10 or decimal system:** the system of numeration based on sets of ten
- **digit:** a single symbol that names a number less than the base of the system
- **place value:** the value assigned to a digit's position within a numeral
- **total value:** the product of the digit and its place value
- **expanded notation:** writing a numeral to show the place value of each digit within the numeral
- **period chart:** a chart showing the periods, such as units, hundreds, and thousands; used to simplify the reading of large numbers

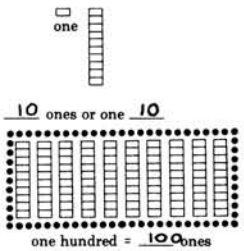
Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, will help you evaluate his understanding of place value.

A Step Further

If students have had no difficulty with the work on the *activity pages*, you may wish to use the following activities to help them further explore place value through millions.

1. For practice in recognizing place value, set up a series of numbers which contain the same digits but are of differing values; for example, 707,770; 707,077; 770,077; 777,007; and 700,777. Have the students place the numerals in ascending order.

1. In base ten, which we call the decimal system, we group in sets of 10. Digits used in base ten (the decimal system) are:
   0, 1, 2, 3, 4, 5, 6, 7, 8, 9

2. 

   10 ones or one 10
   one hundred = 100 ones
   one hundred = 10 sets of tens
   100 = 100 × 1 100 = 10 × 10

PLACE VALUE CHART

Hundreds	Tens	Ones
10 × 10	10	1
100	10	1
3	4	7

3. 347 means 3 hundreds, 4 tens, 7 ones.

4. In the number 347, what is the place value of digit 7? ones digit 3? hundreds and digit 4? tens

5. In the number 347, the total value of digit 3, in the hundreds place, is 3 × 100 or 300.
   The total value of digit 4, in the tens place, is 4 × 10 or 40
   The total value of digit 7, in the ones place, is 7 × 1 or 7.

6. 347 written in expanded notation is 300 + 40 + 7.
   789 written in expanded notation is 700 + 80 + 9.

7. 589
   What is the place value of digit 5? hundreds
   of digit 8? tens of digit 9? ones
   What is the total value of digit 5? 500
   of digit 8? 80 of digit 9? 9
   589 in expanded notation is 500 + 80 + 9.

8. Ten sets of a hundred equals a thousand. How many hundred dollar bills make a thousand dollars?
   10 hundred dollar bills

9. PLACE VALUE CHART

Thousands	Hundreds	Tens	Ones
10 × 10 × 10	10 × 10	10	1
1,000	100	10	1
6	4	7	9

   Each place to the left is 10 times greater.
   In 6,479, what is the place value of digit 6? thousands
   Total value of digit 6? 6,000
   What is the place value of digit 4? hundreds
   Total value of digit 4? 400
   What is the place value of digit 7? tens
   Total value of digit 7? 70
   What is the place value of digit 9? ones
   Total value of digit 9? 9
   6,479 in expanded notation is 6,000 + 400 + 70 + 9.

2

PLACE VALUE CHART

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
10 × 10 × 10 × 10 × 10	10 × 10 × 10 × 10	10 × 10 × 10	10 × 10	10	1
100,000	10,000	1,000	100	10	1
6	5	0	1	2	7

10. How many ten thousands make a hundred thousand? 10
    What is the place value of digit 0 in 650,127? thousands
    What is the total value of digit 5? 50,000
    What is the total value of digit 6? 600,000

PERIOD CHART

Millions			Thousands			Units		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
5	6	8	9	4	3	1	2	0

11. What are the names of the periods shown? millions, thousands, and units

12. In each period, there are three places called hundreds, tens, and ones.

13. What is the place value of digit 6 in the chart? ten millions

14. What is the place value of digit 9? hundred thousands

15. Which digits in the chart are in the millions period? 5, 6, and 8
    The place value of 5 is hundred millions.

3

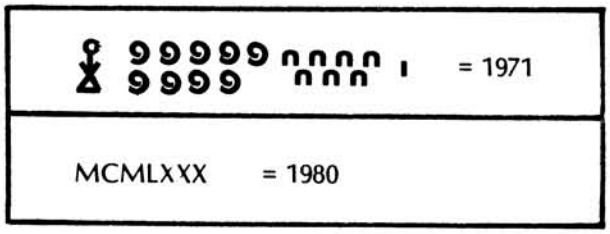
Fill in all the blanks. Use the charts on pages 1, 2, and 3 to help you. Read carefully before you answer.

1. How many tens make a hundred? 10 How many ones make ten? 10
2. In the number 528, what is the place value of digit 5? hundreds
3. Write 729 in expanded notation. 700 + 20 + 9
4. One thousand is ten sets of one hundred
5. In 27,029, what is the place value of digit 7? thousands
6. What is the total value of digit 2 in 27,679? 20,000
7. Write 5,246 in expanded notation. 5000 + 200 + 40 + 6
8. In the number 476,539, the digits 476 are in what period? thousands
9. Write thirty-seven thousand forty-eight as a numeral. 37,048
10. In the number 26,768,234, the digits 26 are in what period? millions
11. What is the place value of digit 4 in 476,539? hundred thousands
12. Write thirty-nine million five hundred eighty-two thousand sixty-four as a numeral. 39,582,064
13. Write one million as a numeral. 1,000,000
14. Two hundreds is how many tens? 20
15. In the number 3,457,984, what is the total value of digit 5? 50,000
16. Write the commas in the numeral 3,4 5 6,7 8 9.
17. Write the word name for this numeral: 310,952,685. three hundred ten million, nine hundred fifty-two thousand, six hundred eighty-five

4

2. To emphasize the importance of place value, introduce your students to ancient systems of numeration. Many of the older systems were cumbersome because the concept of place value was not fully developed. Give students a copy of this chart and ask them to write the modern equivalents for the ancient symbols listed below the chart.

Modern	Egyptian	Roman
1		I
5		V
10		X
50		L
60		LX
100		C
500		D
1000		M



3. Make further use of expanded notation, introduced on the ***audio*** to demonstrate the value of individual digits in numerals. You might use familiar numerals, such as the year or population figures, distances in space, and other five- to nine-digit numerals. Encourage the student to try the method on numbers with place values up to and including millions.

LESSON 2

Addition with Renaming

Approximate time required to complete the lesson: 32 min.

Background Needed

To benefit fully from this lesson, the student should have an understanding of place value through hundreds and know his addition facts through 20 well.

In The Lesson

After guiding the student through several exercises in which ones are renamed as tens and ones, the narrator demonstrates the use of renaming when adding two-digit numbers. The student is given an opportunity to try this and is allowed to correct his own responses. As the lesson progresses, the renaming of three-digit numbers is introduced and used to solve addition problems.

Vocabulary: This term is defined and used on the tape.

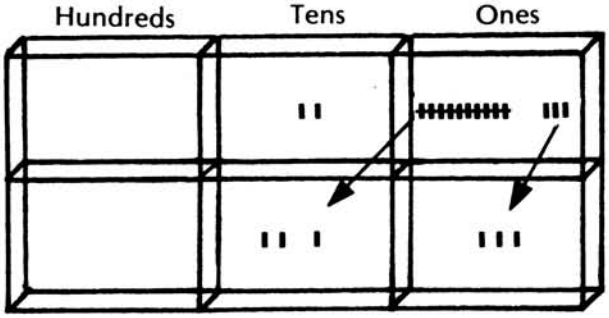
renaming: thinking of or writing a number in a different way

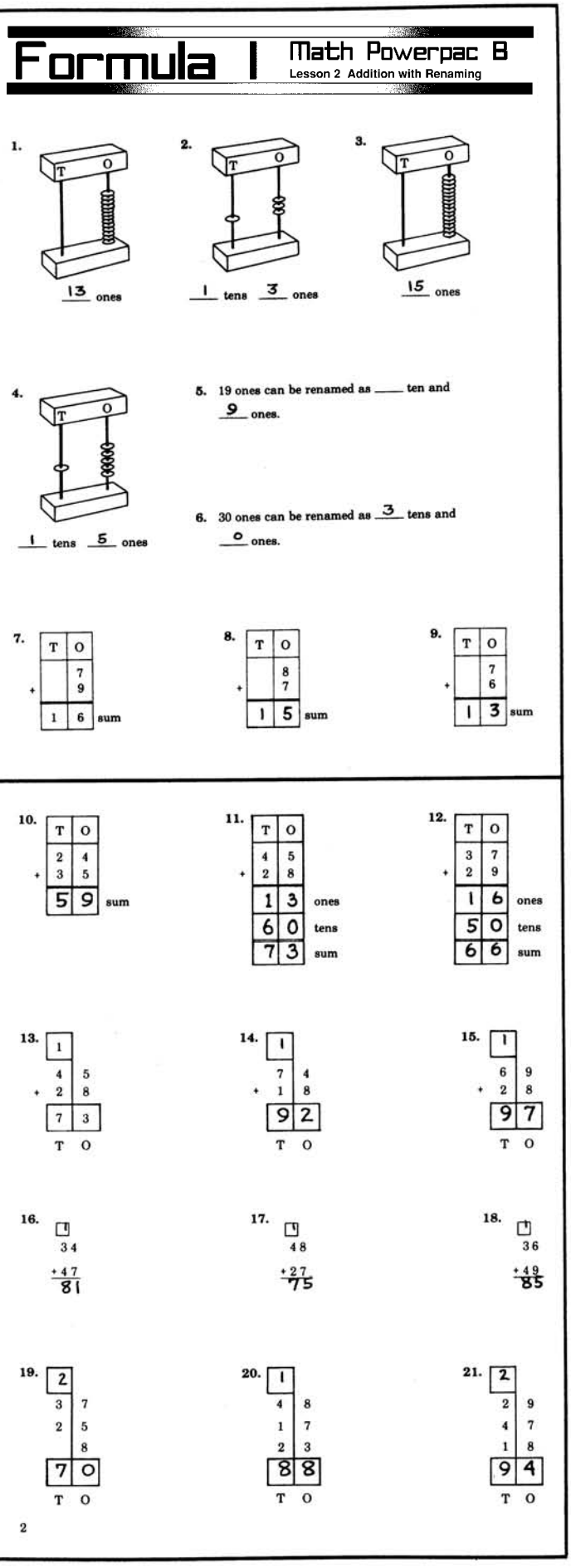
Evaluation: Page 4 of the *activity pages* is completed by the student after the *audio* has finished playing. His performance on this page should give you an indication of his understanding of renaming and its use in addition.

A Step Further

The following activities are included for the purpose of providing able students an opportunity to further explore concepts related to addition and renaming.

1. Construct a "dust abacus," the earliest version of the abacus, by partitioning a shallow cardboard box into six equal sections, filling the sections with sand, and labeling them as shown below. Tally marks may be made in the sand to illustrate renaming.





22.

H	T	O
2	3	4
+ 3	4	5
5	7	9

sum

23.

H	T	O
3	6	4
+ 3	2	5
6	8	9

sum

24.

H	T	O	
4	3	8	
+ 1	4	9	
	1	7	ones
	7	0	tens
5	0	0	hundreds
5	8	7	sum

25.

H	T	O	
3	6	9	
+ 2	2	7	
	1	6	ones
	8	0	tens
5	0	0	hundreds
5	9	6	sum

26.

H	T	O	
4	5	7	
+ 2	8	6	
	1	3	ones
1	3	0	tens
6	0	0	hundreds
7	4	3	sum

27.

H	T	O	
5	8	9	
+ 2	7	6	
	1	5	ones
1	5	0	tens
7	0	0	hundreds
8	6	5	sum

28. 130 = 13 tens = 1 hundred and 3 tens

29. 27 tens = 2 hundreds and 7 tens

30.

2	6	7
+ 3	5	8
6	2	5
H	T	O

31.

4	6	9
+ 2	8	7
7	5	6
H	T	O

32.

5	2	3
+ 3	8	9
9	1	2
H	T	O

3

1. 7 + 9 = 16

2. 27 + 9 = 36

3. 38 + 7 = 45

4. 26 + 8 = 34

5. 27 + 36 = 63

6. 48 + 29 = 77

7. 53 + 39 = 92

8. 82 + 9 = 91

9. 423 + 59 = 482

10. 567 + 79 = 646

11. 639 + 86 = 725

12. 708 + 27 = 735

13. 327 + 286 = 613

14. 648 + 275 = 923

15. 357 + 489 = 846

16. 267 + 484 = 751

17. 48 + 267 + 35 = 350

18. 123 + 409 + 65 = 597

19. 423 + 289 + 109 = 821

20. 507 + 183 + 68 = 758

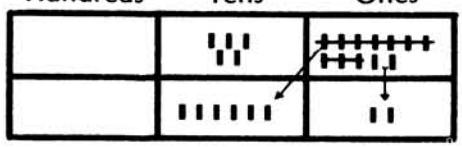
4

Use the following example to introduce "front-end" addition (addition in which the tens are added first). Give several front-end addition problems to students to work; then have them use the dust abacus to check their work.

Problem:

$$\begin{array}{rr} & 37 \\ + & 25 \\ \hline \text{T} & 50 \\ \text{O} & 12 \\ \hline & 62 \end{array}$$

Check:



In checking with the dust abacus, move the marks from the top row to the bottom row and rename as in the illustration.

2. For more practice with addition facts, prepare "crossnumber" charts containing overlapping horizontal and vertical addition problems with one or more missing addends and have the students fill them in.

Example:

2	+	2	+	2	= 6
+		+		+	
2	+	3	+	2	= 7
+		+		+	
2	+	2	+	5	= 9
= 6		= 7		= 9	

3. Introduce Hindu addition, the method of addition used by Hindus in the 12th century. Have students figure out the rules by which this method works and then use the technique in other problems. The method resembles the addition we use today, but it is in horizontal rather than vertical form. First, the digits in each place are added separately. Then, each sum is multiplied by its place value, and the sum of those products is the final sum.

For example:

Problem:

Process:
ones sum = 20 (20 × 1) = 20
tens sum = 14 (14 × 10) = 140
hundreds sum = 2 (1 × 200) = 200

Solution: total sum 360

The final sum is 2 hundreds, 14 tens, and 20 ones or 360.

LESSON 3

Subtraction with Renaming

Approximate time required to complete the lesson: 40 min.

Background Needed

In order to participate successfully in this lesson, the student must know his subtraction facts through 20 and understand place value.

In The Lesson

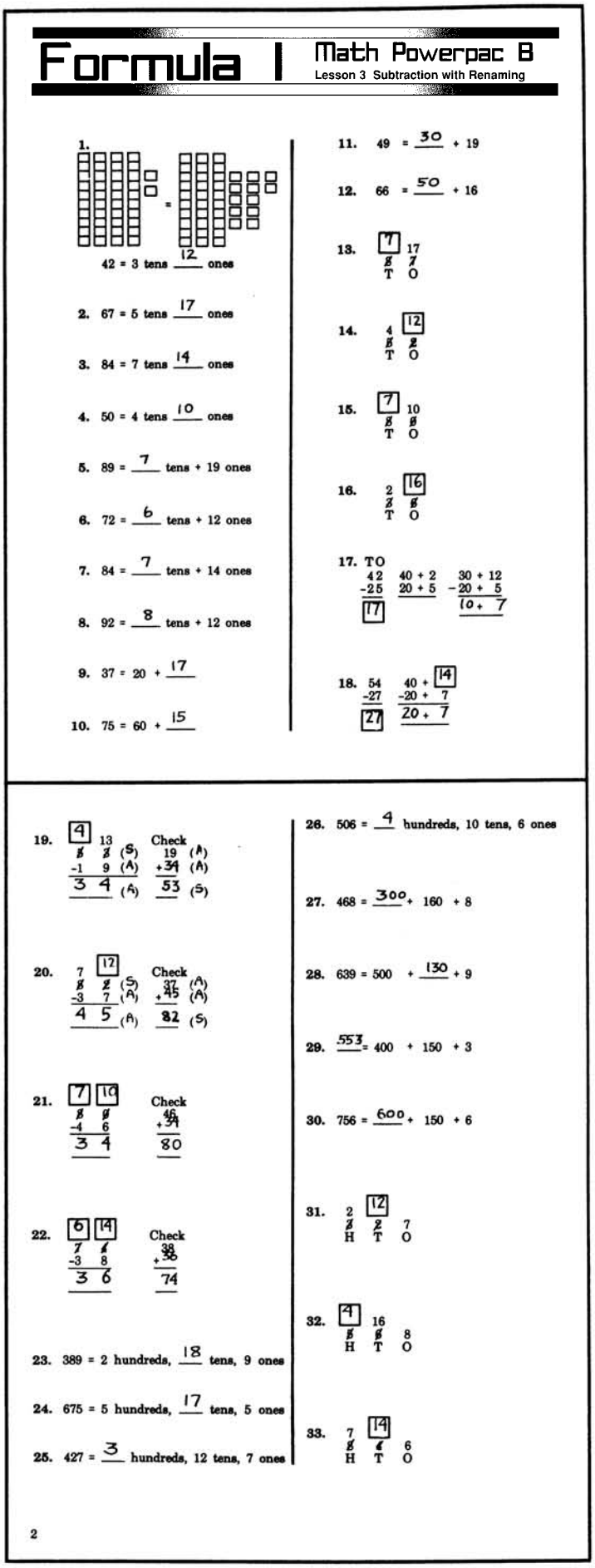
After a brief review of the renaming process, the narrator guides the student through a series of subtraction problems involving two-digit numerals. The student is then led to rename three-digit numerals and, finally, to work subtraction problems with three- digit numerals. Addition is introduced as a means by which the student can check his own work. Responses are checked by the narrator on the first three pages of the *activity pages.*

Evaluation: The student's performance on page 4 of the *activity pages,* which he completes after the *audio* has finished playing, will give you an idea of how well he has understood the use of renaming in subtraction.

A Step Further

If students have had no trouble with the work on the *activity pages,* you may wish to use the following activities to help them explore subtraction with renaming in greater depth.

1. Square subtraction, the procedure illustrated below, offers interesting practice in both renaming and subtraction. For the problem 17 – 8, the 17 is written in the small square, top left, and the 8 in the small square, bottom right. Each number is renamed as two addends. One pair of addends is written across the top and the other pair along the right side. In this case, 17 has been renamed as 10 + 7 and 8 as 6 + 2. The components are then subtracted and put in the appropriate squares. The answer to the original subtraction problem (written in the small square, bottom left) is found by adding the numerals in the top right and bottom left squares, and checked by adding the numerals in the top left and bottom right squares.



34. $\begin{array}{r} \boxed{3}\ 10 \\ 4\ 8\ 6 \\ -2\ 3\ 2 \\ \hline 1\ 7\ 4 \end{array}$ (S) (A) (A)

Check: 232 (A) + 174 (A) = 406 (S)

35. $\begin{array}{r} 7\ \boxed{12} \\ 8\ 2\ 9 \\ -5\ 6\ 4 \\ \hline 2\ 6\ 5 \end{array}$ (S) (A) (A)

Check: 564 (A) + 265 (A) = 829 (S)

36. $\begin{array}{r} \boxed{6}\ 13 \\ 7\ 3\ 8 \\ -4\ 8\ 5 \\ \hline 2\ 5\ 3 \end{array}$

Check: 485 + 253 = 738

37. $\begin{array}{r} 4\ \boxed{10} \\ 5\ 0\ 9 \\ -2\ 4\ 1 \\ \hline 2\ 6\ 8 \end{array}$

Check: 241 + 268 = 509

38. $\begin{array}{r} 4\ \boxed{11}\ \boxed{13} \\ 5\ 2\ 3 \\ -2\ 7\ 8 \\ \hline 2\ 4\ 5 \end{array}$

Check: 278 + 245 = 523

39. $\begin{array}{r} \boxed{7}\ 9\ 11 \\ 8\ 0\ 1 \\ -3\ 4\ 7 \\ \hline 4\ 5\ 4 \end{array}$

Check: 347 + 454 = 801

40. $\begin{array}{r} \boxed{6}\ \boxed{10}\ 14 \\ 7\ 1\ 4 \\ -2\ 6\ 9 \\ \hline 4\ 4\ 5 \end{array}$

Check: 269 + 445 = 714

41. $\begin{array}{r} \boxed{6}\ \boxed{9}\ \boxed{10} \\ 7\ 0\ 0 \\ -3\ 8\ 9 \\ \hline 3\ 1\ 1 \end{array}$

Check: 389 + 311 = 700

3

1. 47 = 3 tens, 17 ones
2. 58 = 4 tens, 18 ones
3. 73 = 6 tens, 13 ones
4. 80 = 7 tens, 10 ones
5. 423 = 3 hundreds, 11 tens, 13 ones
6. 789 = 6 hundreds, 17 tens, 19 ones
7. 600 = 5 hundreds, 9 tens, 10 ones
8. 977 = 9 hundreds, 6 tens, 17 ones

9. $\begin{array}{r} \boxed{7}\ 12 \\ 8\ 2 \\ \text{T}\ \text{O} \end{array}$

10. $\begin{array}{r} 6\ \boxed{19} \\ 7\ 9 \\ \text{T}\ \text{O} \end{array}$

11. $\begin{array}{r} 3\ \boxed{15}\ 18 \\ 4\ 6\ 8 \\ \text{H}\ \text{T}\ \text{O} \end{array}$

12. $\begin{array}{r} 8\ \boxed{10} \\ 9\ 0 \\ \text{T}\ \text{O} \end{array}$

13. $\begin{array}{r} \boxed{4}\ \boxed{11}\ 17 \\ 5\ 2\ 7 \\ \text{H}\ \text{T}\ \text{O} \end{array}$

14. $\begin{array}{r} \boxed{5}\ \boxed{9}\ 10 \\ 6\ 0\ 0 \\ \text{H}\ \text{T}\ \text{O} \end{array}$

15. $\begin{array}{r} 8\ \boxed{11}\ 17 \\ 9\ 2\ 7 \\ \text{H}\ \text{T}\ \text{O} \end{array}$

16. $\begin{array}{r} \boxed{3}\ 11 \\ 4\ 1 \\ -2\ 7 \\ \hline 1\ 4 \end{array}$

Check: 27 + 14 = 41

17. $\begin{array}{r} 5\ \boxed{18} \\ 6\ 8 \\ -3\ 9 \\ \hline 2\ 9 \end{array}$

Check: 39 + 29 = 68

18. $\begin{array}{r} \boxed{6}\ \boxed{10} \\ 7\ 0 \\ -4\ 6 \\ \hline 2\ 4 \end{array}$

Check: 46 + 24 = 70

19. $\begin{array}{r} \boxed{3}\ \boxed{11}\ 13 \\ 4\ 2\ 3 \\ -2\ 7\ 8 \\ \hline 1\ 4\ 5 \end{array}$

Check: 278 + 145 = 423

20. $\begin{array}{r} \boxed{4}\ \boxed{9}\ 0 \\ 5\ 0\ 0 \\ -2\ 3\ 9 \\ \hline 2\ 6\ 1 \end{array}$

Check: 239 + 261 = 500

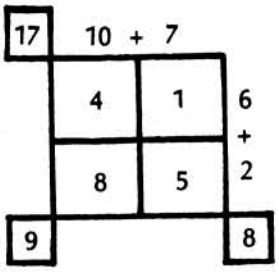
21. $\begin{array}{r} \boxed{7}\ \boxed{11}\ \boxed{17} \\ 8\ 2\ 7 \\ -5\ 3\ 9 \\ \hline 2\ 8\ 8 \end{array}$

Check: 539 + 288 = 827

22. $\begin{array}{r} 43 \\ -\ 28 \\ \hline 15 \end{array}$

23. $\begin{array}{r} 900 \\ -\ 537 \\ \hline 363 \end{array}$

4



17 − 8 = 9

Put the example on the board for students and give them additional sets of subtraction facts to work by this method.

2. As an enrichment activity, show students this variation of renaming with subtraction.

$$\begin{array}{r} 5\not{2}^{\,12} \\ -\ 3\not{7} \\ \hline 15 \end{array}$$

Have them tell you whether the answer was changed when the parts of the problem were renamed. See if they can explain why, if 10 was added to the ones in 52, the 3 tens in 37 was renamed as 4 tens.

Answer: Since 10 was added to the top number, which is a sum, it was necessary to add 10 to the bottom number, an addend, to maintain equality.

$$\begin{array}{r} 52 + 10 = 50 + 12 \\ -\ 37 + 10 = 40 + \ 7 \\ \hline 10 + \ 5 = 15 \end{array}$$

3. Give a number a complement and it will help you solve a subtraction problem. First, find the complement of the number being subtracted. Then, add the complement to the minuend. Mark off the numeral in the hundreds place and the remaining numeral is the answer to the subtraction problem. Examples:

$$\begin{array}{r} 79 \\ -\ 32 \end{array}$$ (The complement of 32 is 68.) $$\begin{array}{r} 68 \\ +\ 79 \\ \hline \not{1}47 \end{array}$$

(Mark off the numeral in the hundreds place.) The answer is 47.

$$\begin{array}{r} 53 \\ -\ 29 \end{array}$$ (The complement of 29 is 71.) $$\begin{array}{r} 71 \\ +\ 53 \\ \hline \not{1}24 \end{array}$$

What is the answer? 24

LESSON 4

Compound Multiplication

Approximate time required to complete the lesson: 45 min.

Background Needed

In this lesson, all the basic multiplication facts are used in solving multiplication problems which include numbers with two or more digits. The student must therefore know his multiplication facts through 81 and understand multiples of 10 and 100.

In The Lesson

The narrator guides the listener in learning how to multiply increasingly larger numbers by building on the multiplication facts he already knows. In the process, he uses arrays, the distributive property of multiplication, and multiples of 10. The long method of multiplication, in which all the partial products are written down, is introduced first. The lesson then proceeds to the use of the short method, in which part of the computation is done in the student's head. The problems in this lesson include two- and three-digit numerals.

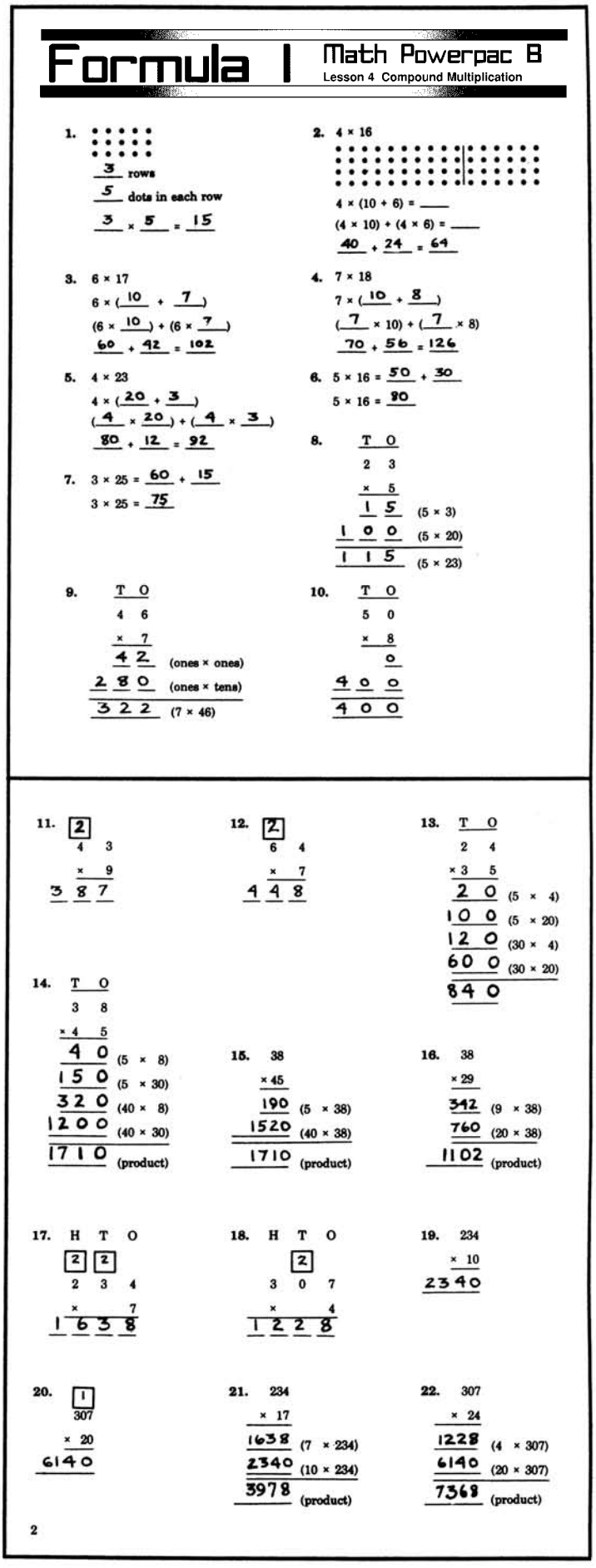
Special Instructions: The student is asked to complete page 3 of the *activity pages* immediately after the *audio* has finished playing. Page 4 consists of enrichment exercises which are both interesting and challenging. To complete the finger multiplication activity, the student must be an able reader in order to comprehend the directions. The student is told by the narrator to do the work on page 4 only if his teacher has instructed him to do so. It is important that you tell the student, before the *audio* begins, whether or not he is to complete the fourth page.

Vocabulary: The following terms are introduced and used by the narrator in this lesson.

compound multiplication: multiplication involving factors with more than one digit

distributive property: the property which allows multiplication by one factor to be distributed over one or more factors which have been renamed as addends

Evaluation: The student's performance on page 3 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate his understanding of two- and three-digit multiplication. If you have directed him to complete the last page of the *activity*



Formula 1 Math Powerpac B
Lesson 4 Compound Multiplication

1. 3 rows, 5 dots in each row; 3 × 5 = 15
2. 4 × 16; 4 × (10 + 6) = ___; (4 × 10) + (4 × 6) = ___; 40 + 24 = 64
3. 6 × 17; 6 × (10 + 7); (6 × 10) + (6 × 7); 60 + 42 = 102
4. 7 × 18; 7 × (10 + 8); (7 × 10) + (7 × 8); 70 + 56 = 126
5. 4 × 23; 4 × (20 + 3); (4 × 20) + (4 × 3); 80 + 12 = 92
6. 5 × 16 = 50 + 30; 5 × 16 = 80
7. 3 × 25 = 60 + 15; 3 × 25 = 75
8. T O: 23 × 5 = 15 (5 × 3) + 100 (5 × 20) = 115 (5 × 23)
9. T O: 46 × 7 = 42 (ones × ones) + 280 (ones × tens) = 322 (7 × 46)
10. T O: 50 × 8 = 0 + 400 = 400
11. 43 × 9 = 387
12. 64 × 7 = 448
13. T O: 24 × 35 = 20 (5 × 4) + 100 (5 × 20) + 120 (30 × 4) + 600 (30 × 20) = 840
14. T O: 38 × 45 = 40 (5 × 8) + 150 (5 × 30) + 320 (40 × 8) + 1200 (40 × 30) = 1710 (product)
15. 38 × 45 = 190 (5 × 38) + 1520 (40 × 38) = 1710 (product)
16. 38 × 29 = 342 (9 × 38) + 760 (20 × 38) = 1102 (product)
17. H T O: 234 × 7 = 1638
18. H T O: 307 × 4 = 1228
19. 234 × 10 = 2340
20. 307 × 20 = 6140
21. 234 × 17 = 1638 (7 × 234) + 2340 (10 × 234) = 3978 (product)
22. 307 × 24 = 1228 (4 × 307) + 6140 (20 × 307) = 7368 (product)

2

1. XXXXXXXXXXXX
XXXXXXXXXXXX
XXXXXXXXXXXX

3 × 13 =

3 × (10 + 3)

(3 × 10) + (3 × 3)

30 + 9 = 39

2. 6 × 17

6 × (10 + 7)

(6 × 10) + (6 × 7)

60 + 42 = 102

3. $\overset{4}{18} \times 6 = 108$

4. $\overset{5}{26} \times 9 = 234$

5. $\overset{5}{67} \times 8 = 536$

6. $80 \times 7 = 560$

7. 23 × 18

184 (8 × 23)

230 (10 × 23)

414 (18 × 23)

8. 42 × 36

252 (6 × 42)

1260 (30 × 42)

1512 (36 × 42)

9. 60 × 52

120

300

3120

10. $\overset{25}{238} \times 7 = 1666$

11. $\overset{1}{436} \times 20 = 8720$

12. $507 \times 9 = 4563$

13. 312 × 28

2496 (8 × 312)

6240 (20 × 312)

8736 (28 × 312)

14. 406 × 39

3654

1218

15,834

15. 530 × 42

1060

2120

22260

3

OTHER WAYS TO MULTIPLY

Duplication Method:

During the early middle ages, multiplication was performed by doubling. Study the example and then try a problem.

31 × 42

(1 × 42 = 42)

(2 × 42 = 84)

(4 × 42 = 168)

(8 × 42 = 336)

(16 × 42 = 672)

31 × 42 = 1302

15 × 54

(1 × 54 = 54)

(2 × 54 = 108)

(4 × 54 = 216)

(8 × 54 = 432)

15 × 54 = 810

63 × 24

(1 × 24 = 24)

(2 × 24 = 48)

(4 × 24 = 96)

(8 × 24 = 192)

(16 × 24 = 384)

(32 × 24 = 768)

63 × 24 = 1512

Finger Multiplication:

Peasants in France used this method for multiplication facts whose factors were larger than 5. To find the product of 9 × 8:

1. *Subtract* 5 from the first factor, 9, and bend down fingers of the left hand to indicate the difference. (9 - 5 = 4).
2. *Subtract* 5 from the second factor, 8, and bend down fingers of the right hand to indicate that difference. (8 - 5 = 3).
3. *Add* the number of bent fingers to find the number of tens. (4 + 3 = 7 tens)
4. *Multiply* the number of unbent fingers on the left hand by the number of unbent fingers on the right hand to get the ones. (1 × 2 = 2). The product is 72.

Now try this method to find the product of 8 × 7.

8 × 7 = 56 .

4

pages, you might offer a subjective comment about the quality of his work in those exercises.

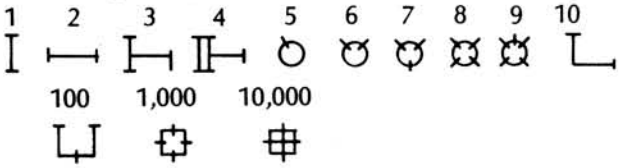
A Step Further

The following activities are suggested to give able students an opportunity to further explore topics related to multiplication.

1. Have students devise their own symbols for a base 10 number system, then translate, and multiply the following numbers in that system.

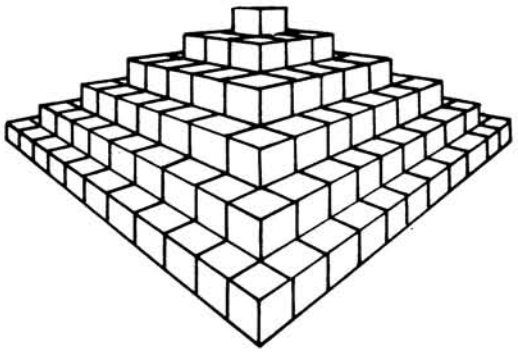
   7 × 2 20 × 14 733 × 41 235 × 220

   You might suggest this as one possible system:



   The first problem solved with this system:

2. Let some of your students try multiplication or addition to solve this puzzle:
   If you were able to view this figure from directly above, how many cubes would be hidden from sight?



Solution: 81 out of 121 hidden in first layer
49 out of 81 hidden in second layer
25 out of 49 hidden in third layer
9 out of 25 hidden in fourth layer
1 out of 9 hidden in fifth layer
None hidden in top layer

165

Alternate Solution: Subtract the number one could see (121) from the total number of cubes (286)
286 - 121 = 165

LESSON 5

Uneven Division — One-Digit Divisor

Approximate time required to complete the lesson: 25 min.

Background Needed

To perform successfully on this lesson, the student should know his division facts through 81.

In The Lesson

The student is first presented with a series of problems in which he must find the greatest missing factor. Then, the number line is used to illustrate uneven division with one-digit divisors. As the lesson continues, the student learns to write the remainder as a fraction and is guided in using multiplication to check his division problems.

Vocabulary: The following term and symbol are introduced and used in this lesson.

remainder: the number left over

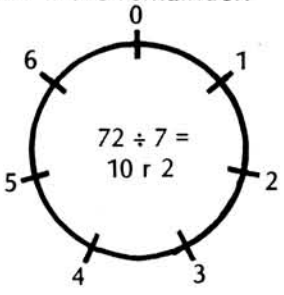
≤: less than or equal to

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should give you an indication of how well he understands the concepts presented.

A Step Further

The following activities are suggested to give able students an opportunity to explore the concepts presented in this lesson in greater depth.

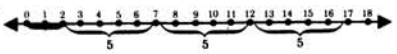
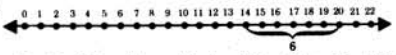
1. For a novel approach to division problems, introduce the circular number line. In solving the division problem 72 ÷ 7, the student starts at zero and moves around the circle clockwise, counting the units as he goes. Each time he passes zero, he puts a tally inside the circle. When he has reached 72 units, he counts the number of tallies in the circle to find the whole quotient. The number of units past zero and less than seven is the remainder.



{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ...}

≤ means less than or equal to

1. □ × 5 ≤ 34
   {0, 1, 2, 3, 4, 5, 6}
2. □ × 8 ≤ 52
   {0, 1, 2, 3, 4, 5, 6}
3. 6 × 5 ≤ 34
4. 6 × 8 ≤ 52
5. 5 × 7 ≤ 40
6. 9 × 8 ≤ 72

7. 
   13 ÷ 4 = 3 r 1 Check: (3 × 4) + 1 = 13
8. 
   17 ÷ 5 = 3 r 2 Check: (3 × 5) + 2 = 17
9. 
   20 ÷ 6 = 3 r 2 Check: (3 × 6) + 2 = 20

10. 6 × 7 ≤ 45
    45 ÷ 7 = 6 r 3
    Check: (6 × 7) + 3 = 45
11. 7 × 5 ≤ 38
    38 ÷ 5 = 7 r 3
    Check: (7 × 5) + 3 = 38
12. 7 × 9 ≤ 67
    67 ÷ 9 = 7 r 4
    Check: (7 × 9) + 4 = 67
13. T O
    2 r 2
    5) 1 2
    1 0
    2
    Check: (2 × 5) + 2 = 12
14. T O
    4 r 3
    7) 3 1
    2 8
    3
    Check: (4 × 7) + 3 = 31
15. T O
    6 r 2
    8) 5 0
    4 8
    2
    Check: (6 × 8) + 2 = 50
16. T O
    7 r 2
    9) 6 5
    6 3
    2
    Check: (7 × 9) + 2 = 65
17. 8 r 6
    9) 7 8
    7 2
    6
    Check: (8 × 9) + 6 = 78

2

18. $5\overline{)31}$ → $6\frac{1}{5}$ (30, 1)

Check: (6 × 5) + 1 = 31

19. $5\overline{)34}$ → $6\frac{4}{5}$ (30, 4)

Check: (6 × 5) + 4 = 34

20. $5\overline{)48}$ → $9\frac{3}{5}$ (45, 3)

Check: (9 × 5) + 3 = 48

21. When a number is divided by 5, the numbers 0, 1, 2, 3, 4 can be remainders.

22. When a number is divided by 7, the numbers 0, 1, 2, 3, 4, 5, 6 can be remainders.

23. When a number is divided by 9, what is the largest number that can be a remainder? 8

Express the remainders as fractions:

24. $8\overline{)59}$ → $7\frac{3}{8}$ (56, 3)

Check: (7 × 8) + 3 = 59

25. $9\overline{)46}$ → $5\frac{1}{9}$ (45, 1)

Check: (5 × 9) + 1 = 46

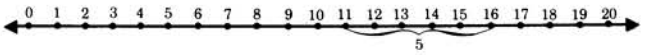
26. $8\overline{)71}$ → $8\frac{7}{8}$ (64, 7)

Check: (8 × 8) + 7 = 71

3

What is the largest whole number which will make each sentence true?

1. $\square \times 7 \leq 40$ → 5
2. $\square \times 8 \leq 58$ → 7
3. $\square \times 9 \leq 63$ → 7
4. $\square \times 6 \leq 51$ → 8
5. $\square \times 7 \leq 62$ → 8
6. $\square \times 4 \leq 38$ → 9



7. 16 ÷ 5 = 3 r 1

Check: (3 × 5) + 1 = 16

8. $7\overline{)40}$ → $5\text{ r }5$ (35, 5)

Check: (5 × 7) + 5 = 40

9. $8\overline{)58}$ → $7\text{ r }2$ (56, 2)

Check: (7 × 8) + 2 = 58

10. $9\overline{)65}$ → $7\text{ r }2$ (63, 2)

Check: (7 × 9) + 2 = 65

11. $6\overline{)57}$ → $9\text{ r }3$ (54, 3)

Check: (9 × 6) + 3 = 57

Write the remainders as fractions:

12. $7\overline{)62}$ → $8\frac{6}{7}$ (56, 6)

Check: (8 × 7) + 6 = 62

13. $4\overline{)37}$ → $9\frac{1}{4}$ (36, 1)

Check: (9 × 4) + 1 = 37

14. $9\overline{)28}$ → $3\frac{1}{9}$ (27, 1)

Check: (3 × 9) + 1 = 28

4

2. Impress students with your "ESP" by playing the "Chinese Remainder" game. Have students follow the steps outlined below:
   a. Write down any number between 1 and 110.
   b. Divide that number by 10 and write the remainder—called the 10-remainder—to the side.
   c. Divide the original number by 11 and write that remainder—called the 11-remainder—to the side.

   When the students have completed the above steps, have one of them tell you his 10-remainder and 11-remainder. To find his original number, multiply his 10-remainder by 11; then multiply his 11-remainder by 100. Add the two products and divide the sum by 110. The remainder will be his original number.

   Example:

Student's part:	Chosen number:	63
	10-remainder:	3
	11-remainder:	8
Your part:	3 × 11 =	33
	8 × 100 =	+ 800
	sum	833

   833 ÷ 110 = 7 remainder 63

3. Able students may enjoy looking for patterns. Present the following number sentence:

   $(\square \times 9) \div (\square \times 3) = n$

   Directions:
   Replace the frames with 3 and find the quotient.
   Replace the frames with 4 and find the quotient.
   Replace the frames with 7 and find the quotient.

   What is the pattern? When the dividend and the divisor are multiplied by the same number, the quotient is unchanged.

4. Challenge students to find patterns in the following:

546 ÷ 13 = 42	546 ÷ 26 = 21
6958 ÷ 49 = 142	6958 ÷ 142 = 49
225 ÷ 25 = 9	450 ÷ 25 = 18
1160 ÷ 40 = 29	2320 ÷ 40 = 58
330 ÷ 30 = 11	330 ÷ 15 = 22
336 ÷ 14 = 24	336 ÷ 7 = 48

LESSON 6

Long Division — Two-Digit Divisor

Approximate time required to complete the lesson: 45 min.

Background Needed

This lesson deals with the division of up to five-digit numbers. Therefore, it is important that the student have a good understanding of place value before starting the lesson. He should also have a good command of basic division facts and know how to use multiplication to check division.

In The Lesson

In introducing the listener to long division with two-digit divisors, the narrator demonstrates that division is the easiest operation to use in solving certain types of problems. He teaches the student how to estimate quotients and uses several problems as examples to guide him through the steps necessary to find the answer. Then, the student is asked to try some problems on his own. The problems range in difficulty from dividends in the tens to dividends in the ten thousands, with place value emphasized throughout the lesson. The student is required to check his division problems with multiplication.

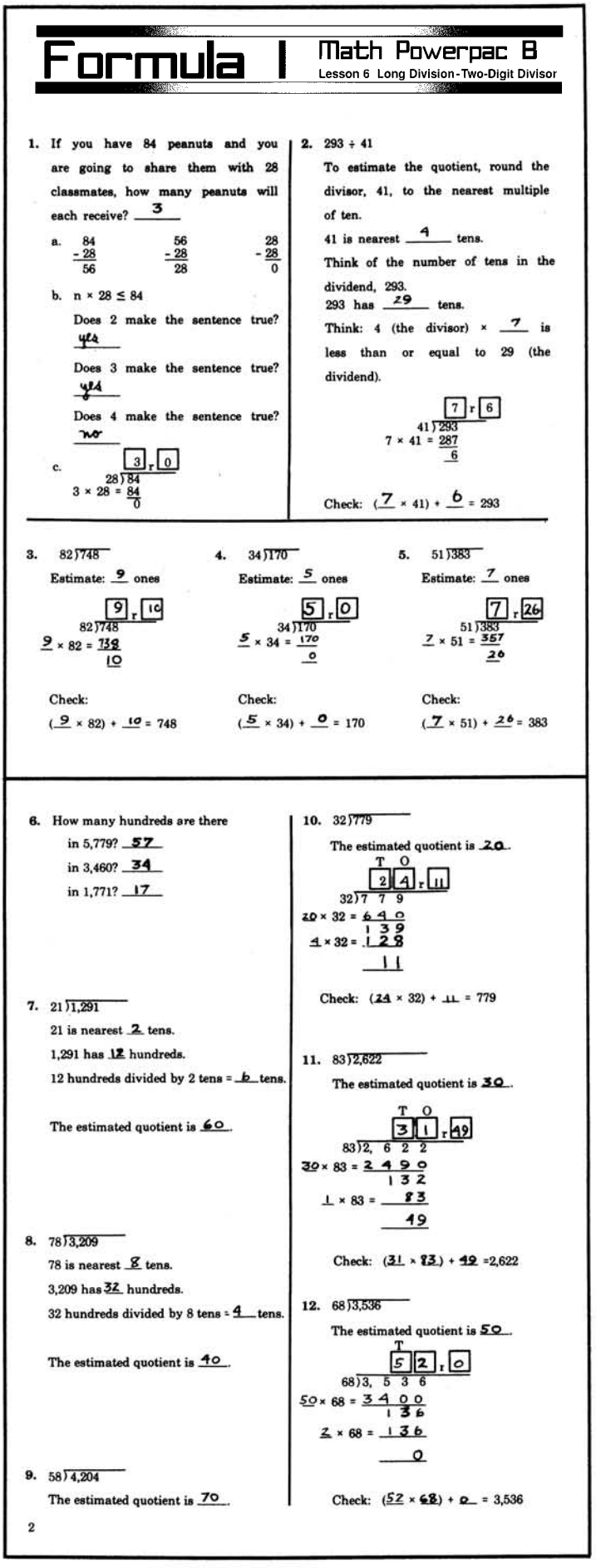
Special Instructions: Each student will need a sheet of scratch paper on which to do computations during the lesson.

Evaluation: The student completes page 4 of the *activity pages* after the *audio* has finished playing. His performance on this page will help you assess his understanding of the long division method presented on the tape.

A Step Further

The following activities will give students an opportunity to expand their knowledge of the topics presented in this lesson.

1. The *audio* directs the listener to check his division problems with this method: Quotient times divisor (plus remainder) equals dividend. For variety, have students try checking some of the division problems on their *activity pages* by casting out nines. In all steps, digits should be added to the smallest sum equal to or less than 9.



Formula 1 Math Powerpac B
Lesson 6 Long Division-Two-Digit Divisor

1. If you have 84 peanuts and you are going to share them with 28 classmates, how many peanuts will each receive? 3

a. 84 − 28 = 56; 56 − 28 = 28; 28 − 28 = 0

b. n × 28 ≤ 84
Does 2 make the sentence true? yes
Does 3 make the sentence true? yes
Does 4 make the sentence true? no

c. 28)84 → 3 r 0
3 × 28 = 84
0

2. 293 ÷ 41
To estimate the quotient, round the divisor, 41, to the nearest multiple of ten.
41 is nearest 40 tens.
Think of the number of tens in the dividend, 293.
293 has 29 tens.
Think: 4 (the divisor) × 7 is less than or equal to 29 (the dividend).

41)293 → 7 r 6
7 × 41 = 287
6

Check: (7 × 41) + 6 = 293

3. 82)748
Estimate: 9 ones
9 × 82 = 738
10
Check: (9 × 82) + 10 = 748

4. 34)170
Estimate: 5 ones
5 × 34 = 170
0
Check: (5 × 34) + 0 = 170

5. 51)383
Estimate: 7 ones
7 × 51 = 357
26
Check: (7 × 51) + 26 = 383

6. How many hundreds are there
in 5,779? 57
in 3,460? 34
in 1,771? 17

7. 21)1,291
21 is nearest 2 tens.
1,291 has 12 hundreds.
12 hundreds divided by 2 tens = 6 tens.
The estimated quotient is 60.

8. 78)3,209
78 is nearest 8 tens.
3,209 has 32 hundreds.
32 hundreds divided by 8 tens = 4 tens.
The estimated quotient is 40.

9. 58)4,204
The estimated quotient is 70.

10. 32)779
The estimated quotient is 20.
32)779 → T: 2 r 11; O: 4
20 × 32 = 640
4 × 32 = 128
11
Check: (24 × 32) + 11 = 779

11. 83)2,622
The estimated quotient is 30.
83)2,622 → T: 3 r 49; O: 1
30 × 83 = 2490
132
1 × 83 = 83
49
Check: (31 × 83) + 49 = 2,622

12. 68)3,536
The estimated quotient is 50.
68)3,536 → T: 5 r 0; O: 2
50 × 68 = 3400
136
2 × 68 = 136
0
Check: (52 × 68) + 0 = 3,536

2

13. T O: 2 7 r 0

91)2 4 5 7

20 × 91 = 820

637

7 × 91 = 637

0

Check: (27 × 91) + 0 = 2457

14. T O: 7 5 r 8

48)3 6 0 8

70 × 48 = 3360

248

5 × 48 = 240

8

Check: (75 × 48) + 8 = 3608

15. T O: 2 6 r 21

31)8 2 7

20 × 31 = 620

207

6 × 31 = 186

21

Check: (26 × 31) + 21 = 827

16. In 31,415 there are 31 thousands.
In 55,251 there are 55 thousands.

7. To divide 38,148 by 57, first estimate the quotient.
57 is nearest 6 tens.
38,148 has about 38 thousands.
38 thousands ÷ by 6 tens = 6 hundreds.

H T O: 6 6 9 r 15

57)3 8, 1 4 8

600 × 57 = 34200

3948

60 × 57 = 3420

528

9 × 57 = 513

15

18. H T O: 3 5 3 r 0

51)1 8, 0 0 3

300 × 51 = 15300

2703

50 × 51 = 2550

153

3 × 51 = 153

0

3

For problems 1-6, just estimate the quotients.

1. 43)221
22 tens ÷ 4 tens = 5 ones.

2. 83)776
77 tens ÷ 8 tens = 9 ones.

3. 78)3209
32 hundreds ÷ 8 tens = 4 tens.

4. 67)2909
29 hundreds ÷ 7 tens = 4 tens.

5. 82)19,301
19 thousands ÷ 8 tens = 2 hundreds.

6. 56)31,415
31 thousands ÷ 6 tens = 5 hundreds.

Estimate the quotient, then divide and check.

7. 5 r 6

43)2 2 1

5 × 43 = 215

6

Check: (5 × 43) + 6 = 221

8. T O: 4 1 r 11

78)3 2 0 9

40 × 78 = 3120

89

1 × 78 = 78

11

Check: (41 × 78) + 11 = 3209

9. T O: 4 3 r 28

67)2 9 0 9

40 × 67 = 2680

229

3 × 67 = 201

28

Check: (43 × 67) + 28 = 2909

10. H T O: 5 6 0 r 55

56)3 1, 4 1 5

500 × 56 = 28000

3415

60 × 56 = 3360

55

0 × 56 = 0

55

Check: (560 × 56) + 55 = 31,415

4

Example: 7894 ÷ 23 = 343 r 5

1st step: add the digits in the quotient, 343
3 + 4 + 3 = 10 = 1 + 0 = 1

2nd step: add the digits in the divisor, 23
2 + 3 = 5

3rd step: multiply the divisor by the quotient
5 × 1 = 5

4th step: add the remainder of the original problem to the result in step 3
5 + 5 = 10 = 1 + 0 = 1

5th step: add the digits in the dividend
7 + 8 + 9 + 4 = 28 = 10 = 1 + 0 = 1

Check: 1 = 1
The check number for the divisor times the check number for the quotient plus the remainder is equal to the check number for the dividend. We can be reasonably sure the problem is correct.

2. The Rhind Papyrus, which is about 3,600 years old, displays the "doubling" technique of division used by ancient Egyptians. In the example below, the divisor, 13, is multiplied in turn by factors which are successively doubled. In the problem, 727 ÷ 13, 32 is the largest factor used because multiplying the next double, 64, times 13 would yield a product larger than the dividend. When the multiplication has been accomplished, the products are added (from bottom to top) to find the combination that yields the greatest sum less than or equal to the dividend. The difference between that sum and the dividend is the remainder. To find the quotient, add the doubled factors of the products used. After you have explained the example to students, have them try a few "ancient Egyptian" division problems on their own.

Example:

727 ÷ 13
1 × 13 = 13
2 × 13 = 26
4 × 13 = 52
8 × 13 = 104
16 × 13 = 208
32 × 13 = 416

727 = 416 + 208 + 52 + 26 + 13 + remainder 12
= 32(13) + 16(13) + 4(13) + 2(13) + 1(13) + remainder 12

32 + 16 + 4 + 2 + 1 = 55

Answer: 727 ÷ 13 = 55 remainder 12

Formula 1 Math Powerpac C

LESSON 1

Prime and Composite Numbers

Approximate time required to complete the lesson: 30 min.

Background Needed

To gain full benefit from this lesson, the student should know his basic multiplication facts and understand the terms used in discussing sets.

In The Lesson

By listing the factors of several whole numbers, the student is led to discover that certain numbers, known as prime numbers, have only themselves and one as factors. Building upon this knowledge, the narrator guides the student through a series of exercises involving prime and composite numbers, twin prime numbers, and relatively prime numbers.

Vocabulary: The following terms are introduced and used on the *audio.*

prime number: a whole number which has only itself and one as factors

composite number: a whole number which has at least one factor in addition to itself and one

twin primes: two prime numbers with a difference of two

relatively prime numbers: numbers which have no common factors other than one

Evaluation: **The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should give you an indication of how well he understands prime and composite numbers.**

A Step Further

The following activities will give your able students an opportunity to further explore the mathematical concepts introduced in this lesson.

1. Euler's (pronounced "oilers") Theorem states that every prime number—except 2—can be expressed as either 4n + 1 or 4n – 1, when *n* is a whole number. Have students use Euler's Theorem to find whether the numbers below are "possibly prime" or "not prime." Remind them that not every number expressed by 4n ± 1 is prime, but that all prime numbers fit the formula.

Formula 1 Math Powerpac C
Lesson 1 Prime and Composite Numbers

1. $2 \times 4 = 8$
   $4 \times 2 = 8$
   $8 \times 1 = 8$
   $1 \times 8 = 8$
   What numbers are factors of 8?
   1, 2, 4, 8

2. $3 \times 4 = 12$
   $4 \times 3 = 12$
   $6 \times 2 = 12$
   $2 \times 6 = 12$
   $12 \times 1 = 12$
   $1 \times 12 = 12$
   What numbers are factors of 12?
   1, 2, 3, 4, 6, 12

3. $7 \times 1 = 7$
   $1 \times 7 = 7$
   What numbers are factors of 7?
   1, 7

4. $1 \times 5 = 5$
   $5 \times 1 = 5$
   What numbers are factors of 5?
   1, 5

5.

	(2)	(3)	4	(5)	6	(7)	8	9	10
(11)	12	(13)	14	15	16	(17)	18	(19)	20
21	22	(23)	24	25	26	27	28	(29)	30
(31)	32	33	34	35	36	(37)	38	39	40
(41)	42	(43)	44	45	46	(47)	48	49	50

6. The prime numbers between 1 and 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

7. How many of the prime numbers are even? 1

8. Name all the composite numbers between 3 and 17. 4, 6, 8, 9, 10, 12, 14, 15, 16

9. Name an odd number less than 10 that is *not prime.* 9

10. Name the odd numbers between 20 and 40 that are *not prime.*
    21, 25, 27, 33, 35, 39

11. Circle the numbers below that are *composite* numbers.
    (21) 13, (34) (25) 17, (48) 37

12. Circle the numbers below that are *prime* numbers.
    33, (19) 27, (31) (41) 25, 49, (23)

13. Write the pairs of *twin prime numbers* less than 47.
    3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43

2

14. Numbers with only two factors, themselves and one, are called prime numbers.

15. The only *even* number that is prime is 2.

16. The set of factors of 3 is {3, 1}.
The set of factors of 10 is {1, 5, 2, 10}.
{3, 1} ∩ {1, 10, 2, 5} = {1}. The common factor is 1.
The numbers 3 and 10 are relatively prime because their only common factor is one.

When two numbers have no common factor other than 1, the numbers are relatively prime.

17. The set of factors of 8 is {1, 8, 2, 4}.
The set of factors of 9 is {1, 9, 3}.
{1, 8, 2, 4} ∩ {1, 9, 3} = {1}.
This means 8 and 9 are relatively prime.

18. Write yes or no below each pair of relatively prime numbers.

a. 5 and 15	b. 7 and 20	c. 6 and 11	d. 8 and 14
no	yes	yes	no

3

1. Write the set of prime numbers greater than 0 and less than 10. {2, 3, 5, 7}
2. Write the set of composite numbers greater than 0 and less than 13. {4, 6, 8, 9, 10, 12}
3. What prime number is even? 2
4. Are all odd numbers prime? no Give an example to prove your answer. 9
5. Why is 5 the only prime number that ends in 5? because all the others are multiples of 5
6. Is 39 prime or composite? composite Why? 3 × 13
7. Is 23 prime or composite? prime Why? it has only factors of itself and one.
8. Name one pair of twin primes. 5 and 3
9. Write the following even numbers as the sum of two prime numbers.
12 = 7+5 16 = 11+5 or 3+13
24 = 11+13 or 19+5 or 7+17 20 = 13+7 or 3+17
10. Write the following odd numbers as the sum of three prime numbers. (other combinations possible)
15 = 3+7+5 23 = 13+3+7 21 = 7+7+7
11. Write "Yes" or "No" to show which sets of numbers are relatively prime.

a. 5 and 14	b. 9 and 12	c. 8 and 15	d. 7 and 20
yes	no	yes	yes

12. The set of factors of 4 is {2, 1, 4}
The set of factors of 9 is {1, 3, 9}
The common factor is 1.

4

a. **159** "possibly prime" according to Euler's Theorem, but further study would reveal that it is not a prime number because it has 53, 3, 159, and 1 as factors

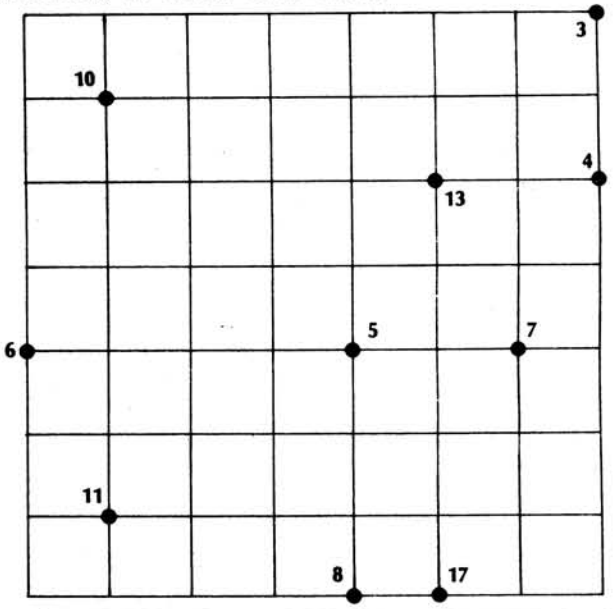
b. **217** "possibly prime" but has 7, 31, 217, and 1 as factors

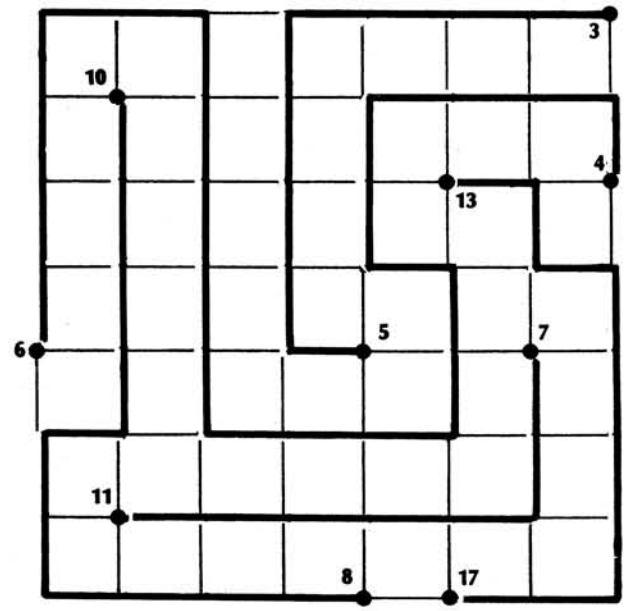
c. **314** (not prime)

d. **167** "possibly prime" and further study would indicate that it is a prime number

e. **224** (not prime)

2. There are 10 points in this maze, each labeled with a prime or a composite number. Students must draw a line to join each prime number to its "prime mate"—the next prime number in sequence—and each composite number to its "composite mate"—the next composite number in sequence—by following the lines on the grid. The student's lines cannot cross, intersect, or touch each other.





LESSON 2

Prime Factorization

Approximate time required to complete the lesson: 40 min.

To benefit fully from this lesson, the student must know how to determine whether a number is prime or composite and how to factor a number. (Successful completion of Lesson 1 should be adequate preparation for this lesson.

In The Lesson

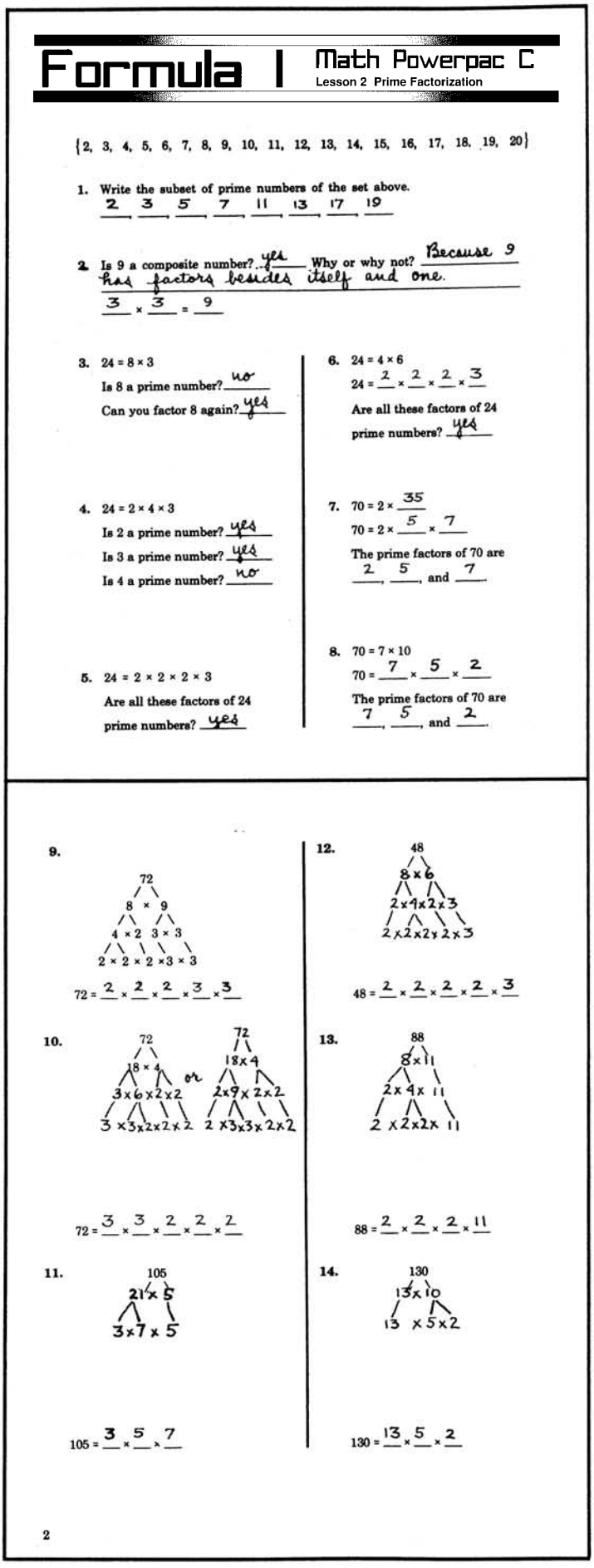
After a brief review of prime and composite numbers, the narrator explains prime factorization. The "factor tree" is introduced and the student is guided to the discovery that, even though the first factors he chooses may vary, the prime factors of any number are always the same.

Evaluation: **The student's performance on page 4 of the *activity pages* which he completes after the *audio* has finished playing, should indicate his understanding of prime factorization.**

A Step Further

The following activities will provide able students an opportunity to expand their understanding of topics related to prime factorization.

1. Let students determine whether or not a number is prime by dividing that number by prime number divisors. The student should begin with 2 and continue to divide by prime numbers in sequence until further whole number division is impossible. The prime factors are circled in this example:
   ② 184
   ② 42
   ③ 21
   ⑦
2. Present a "new" numeration system in which the digits are { 1, 4, 7, 10, 13, 16, 19, 22... }. In this system, 16 is a composite number because it has the factor 4 in addition to itself and 1. Twenty-two is a prime number in this system because its only factors are itself and 1. (Eleven and two are not a part of the new number system.) Ask students the following questions about this "new" system.



Formula 1 Math Powerpac C
Lesson 2 Prime Factorization

{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

1. Write the subset of prime numbers of the set above.
   2, 3, 5, 7, 11, 13, 17, 19

2. Is 9 a composite number? yes Why or why not? Because 9 has factors besides itself and one.
   3 × 3 = 9

3. 24 = 8 × 3
   Is 8 a prime number? no
   Can you factor 8 again? yes

4. 24 = 2 × 4 × 3
   Is 2 a prime number? yes
   Is 3 a prime number? yes
   Is 4 a prime number? no

5. 24 = 2 × 2 × 2 × 3
   Are all these factors of 24 prime numbers? yes

6. 24 = 4 × 6
   24 = 2 × 2 × 2 × 3
   Are all these factors of 24 prime numbers? yes

7. 70 = 2 × 35
   70 = 2 × 5 × 7
   The prime factors of 70 are 2, 5, and 7.

8. 70 = 7 × 10
   70 = 7 × 5 × 2
   The prime factors of 70 are 7, 5, and 2.

9. 72 → 8 × 9 → 4 × 2, 3 × 3 → 2 × 2 × 2 × 3 × 3
   72 = 2 × 2 × 2 × 3 × 3

10. 72 → 3 × 6 × 2 × 2 → 3 × 3 × 2 × 2 × 2 or 72 → 18 × 4 → 2 × 9 × 2 × 2 → 2 × 3 × 3 × 2 × 2
    72 = 3 × 3 × 2 × 2 × 2

11. 105 → 21 × 5 → 3 × 7 × 5
    105 = 3 × 5 × 7

12. 48 → 8 × 6 → 2 × 4 × 2 × 3 → 2 × 2 × 2 × 2 × 3
    48 = 2 × 2 × 2 × 2 × 3

13. 88 → 8 × 11 → 2 × 4 × 11 → 2 × 2 × 2 × 11
    88 = 2 × 2 × 2 × 11

14. 130 → 13 × 10 → 13 × 5 × 2
    130 = 13 × 5 × 2

2

15. All even numbers have 2 as a factor.

16. All numbers that end in 0 or 5 have 5 as a factor.

17. Add the digits in 432. The sum is 9.

18. Add the digits in 7425. The sum is 18.

19. Circle the numbers that have 9 as a factor.

(27); 128; (3,276); (98,784); 247

20. Write 210 in prime factorization.

210 = 21 × 10

210 = 7 × 3 × 5 × 2

21. Write 396 in prime factorization.

396 = 9 × 44

396 = 3 × 3 × 4 × 11

396 = 3 × 3 × 2 × 2 × 11

22. Write 145 in prime factorization.

145 = 5 × 29

23. Write 140 in prime factorization.

140 = 5 × 28

140 = 5 × 2 × 2 × 7

3

1. List the prime numbers less than 15.

2, 3, 5, 7, 11, 13

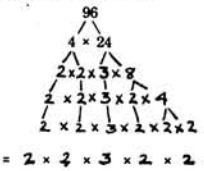
2. Circle the number that has 9 as a factor.

2351 or (4563)

3. Because 175 ends in 5, you know that 5 is a factor.

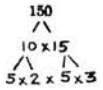
4. Because 48 is an even number, you know that 48 has 2 as a factor.

5. Complete this factor tree.



96 = 2 × 2 × 3 × 2 × 2 × 2

6. Complete the factor tree.



150 = 5 × 2 × 5 × 3

7. Write 54 in prime factorization.

54 = 2 × 27

54 = 2 × 3 × 3 × 3

8. Write 110 in prime factorization.

110 = 11 × 10

110 = 11 × 2 × 5

9. Write 81 in prime factorization.

81 = 9 × 9

81 = 3 × 3 × 3 × 3

10. Write 125 in prime factorization.

125 = 5 × 25

125 = 5 × 5 × 5

4

Is 100 in this system? (Yes)
If so, is it composite or prime? (Composite)
Is 40 in the system? (Yes)
If so, is it prime or composite? (Composite)

You may wish to have students think of additional questions to ask each other about the system.

3. Ancient Greek mathematicians, in studying factors, discovered four categories of numbers based on the sums of their factors. (The Greeks did not include the number itself as a factor.) Present the following puzzlers to your students and let them develop their own definitions.

Here are some *deficient* numbers. Add their factors and find out why they have been given this name.

8 1 + 2 + 4 = 7
10 1 + 5 + 2 = 8
14 1 + 7 + 2 = 10

The sum of deficient number's factors is less than that number.

Can you name others? (15, 9, 21)

Figure out the rule that determines *abundant* numbers.

18 1 + 6 + 3 + 2 + 9 = 21
12 4 + 3 + 1 + 6 + 2 = 16
16 1 + 4 + 4 + 8 + 2 = 19

The sum of an abundant number's factors is greater than that number.

Can you name others? (24, 48, 20)

These are *perfect* numbers. What is the rule governing them?

6 1 + 3 + 2 = 6
28 1 + 7 + 4 + 14 + 2 = 28

The sum of a perfect number's factors is equal to that number.

Find and add all the factors of each of these numbers and explain why they are called *amicable* numbers.

284 1 + 2 + 4 + 71 + 142 = 220
220 1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284

Amicable numbers are a pair of numbers in which the factors of the first add up to the second and the factors of the second add up to the first.

LESSON 3

Least Common Multiples

Approximate time required to complete the lesson: 25 min.

A knowledge of prime and composite numbers, and prime factorization, in addition to mastery of basic multiplication facts, is essential to successful participation in this lesson. It is therefore recommended that the student complete Lesson 1, *Prime and Composite Numbers,* and Lesson 2, *Prime Factorization,* before attempting this lesson.

In The Lesson

Through a brief review of multiples, the narrator leads the student toward recognition and understanding of least common multiples. Two methods of finding least common multiples are presented. The first involves listing sets of multiples for each factor (to determine intersection of the sets). The second makes use of prime factorization. The review of these concepts prepares the student to work with fractions.

Vocabulary: The following term is introduced and used by the narrator in this lesson.

least common multiple: the lowest multiple of two or more whole numbers

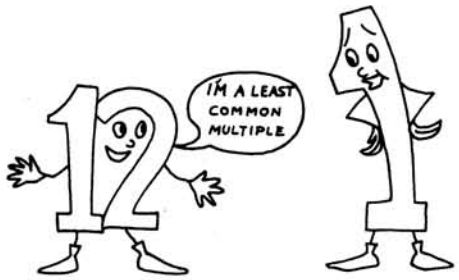
Evaluation: **The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should give you an indication of his ability to find least common multiples.**

A Step Further

The following activities will provide students with an opportunity to practice finding least common multiples.

1. Let students put their skill to work by finding the least common multiple in this story problem.

   Two clocks are started at 12 o'clock. One clock has an alarm that goes off every 12 minutes and the other has an alarm that goes off every 20 minutes. What time will it be when both alarms first go off together?



Formula | Math Powerpac C
Lesson 3 Least Common Multiples

1. The set of multiples of 8, less than or equal to 88:
   {8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88}

2. The set of multiples of 6, less than or equal to 72:
   {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72}

3. What are the *common* multiples of 8 and 6? 24, 48, 72

4. What is the *least* common multiple of 8 and 6? 24

5. Is 8 a factor of 24? yes
   Is 6 a factor of 24? yes
   Is 24 a multiple of 8 and 6? yes

6. Write 8 in prime factorization:
   8 = 2 × 2 × 2

7. Write 6 in prime factorization:
   6 = 2 × 3

8. 8 = 2 × 2 × 2
   6 = 2 × 3
   LCM = 2 × 2 × 2 × 3 or 24

9. The set of multiples of 12, less than or equal to 84:
   {12, 24, 36, 48, 60, 72, 84}

10. The set of multiples of 9, less than or equal to 99:
    {9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99}

11. What are the *common* multiples of 12 and 9? 36, 72

12. What is the *least* common multiple of 12 and 9? 36

13. Is 36 a multiple of 12 and 9? yes

14. Write 12 in prime factorization. 12 = 2 × 2 × 3

15. Write 9 in prime factorization. 9 = 3 × 3

16. 12 = 2 × 2 × 3
    9 = 3 × 3
    LCM = 2 × 2 × 3 × 3 or 36

17. 12 × 3 = 36
    9 × 4 = 36
    Is 36 a multiple of 12 and 9? yes

2

18. Find the LCM of 40 and 24.

$40 = 2 \times 2 \times 2 \times 5$

$24 = 2 \times 2 \times 2 \times 3$

LCM = $2 \times 2 \times 2 \times 5 \times 3$ or 120

19. $40 \times 3 = 120$

$24 \times 5 = 120$

Is 120 a multiple of 24 and of 40? yes

20. Find the LCM of 55 and 70.

$55 = 5 \times 11$

$70 = 7 \times 2 \times 5$

LCM = $5 \times 11 \times 7 \times 2$ or 770

21. Is 770 a multiple of:

70? yes 55? yes

22. Find the LCM of 72 and 48.

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$48 = 2 \times 2 \times 2 \times 2 \times 3$

LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 2$ or 144

23. Find the LCM of 110 and 44.

$110 = 11 \times 2 \times 5$

$44 = 2 \times 2 \times 11$

LCM = $11 \times 2 \times 5 \times 2$ or 220

24. Find the LCM of 27 and 63.

$27 = 3 \times 3 \times 3$

$63 = 3 \times 3 \times 7$

LCM = $3 \times 3 \times 3 \times 7$ or 189

3

1. The set of multiples of 9, less than or equal to 72:

{9, 18, 27, 36, 45, 54, 63, 72}

2. The set of multiples of 6, less than or equal to 72:

{6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72}

3. What is the least common multiple of 9 and 6? 18

4. Is 72 a common multiple of 9 and 6?

yes

5. Find the least common multiple of 6 and 9 using prime factorization.

$6 = 2 \times 3$

$9 = 3 \times 3$

LCM = $2 \times 3 \times 3$ or 18

6. $6 \times 6 = 36$

$9 \times 4 = 36$

Is 36 a multiple of 9 and 6?

yes

7. Find the least common multiple of 18 and 45 using the prime factorization method.

$18 = 3 \times 3 \times 2$

$45 = 3 \times 3 \times 5$

LCM = $3 \times 3 \times 2 \times 5$ or 90

8. Find the LCM of 36 and 40 using prime factorization method.

$36 = 2 \times 2 \times 3 \times 3$

$40 = 2 \times 2 \times 2 \times 5$

LCM = $2 \times 2 \times 3 \times 3 \times 2 \times 5$ or 360

9. Find the LCM of 15 and 35.

$15 = 3 \times 5$

$35 = 7 \times 5$

LCM = $3 \times 7 \times 5$ or 105

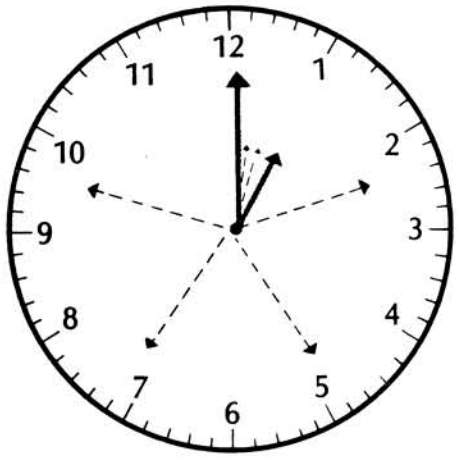
10. $24 = 2 \times 2 \times 2 \times 3$

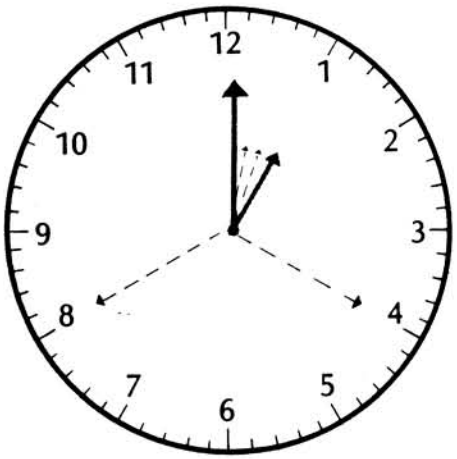
$36 = 2 \times 2 \times 3 \times 3$

LCM = $2 \times 2 \times 2 \times 3 \times 3$ or 72

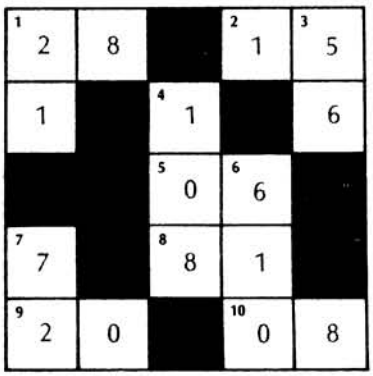
4

The student should find the least common multiple of 12 and 20 to arrive at the answer. The LCM is 60, so the two alarms will coincide at 1 o'clock, 60 minutes after the clocks are started.





2. Let your crossword puzzle fans try this novel "cross-LCM" puzzle.



Across

1. the LCM of 4 and 7
2. the LCM of 3 and 5
5. the LCM of 2 and 3
8. the LCM of 3 and 81
9. the LCM of 5 and 4
10. the LCM of 2 and 8

Down

1. the LCM of 3 and 7
3. the LCM of 7 and 8
4. the LCM of 4 and 27
6. the LCM of 10 and 61
7. the LCM of 8 and 18

LESSON 4

Equivalent Fractions

Approximate time required to complete the lesson: 38 min.

To benefit fully from this lesson, the student should understand the meaning of fractions and be able to write fractional numerals. If his skill in these areas is questionable, it is suggested that you precede the use of this lesson with a review of fractions.

In The Lesson

Divided congruent figures and number lines are used in this lesson to establish recognition and understanding of equivalent fractions. The student is first taught to rename the whole number, one, in fraction form. Then he learns to use the identity element to find equivalent fractions. The lesson continues with an explanation of writing fractions in simplest form. The student is given ample opportunity to practice this skill in a series of exercises on the *activity pages.*

Vocabulary: The following term is introduced and used by the narrator.

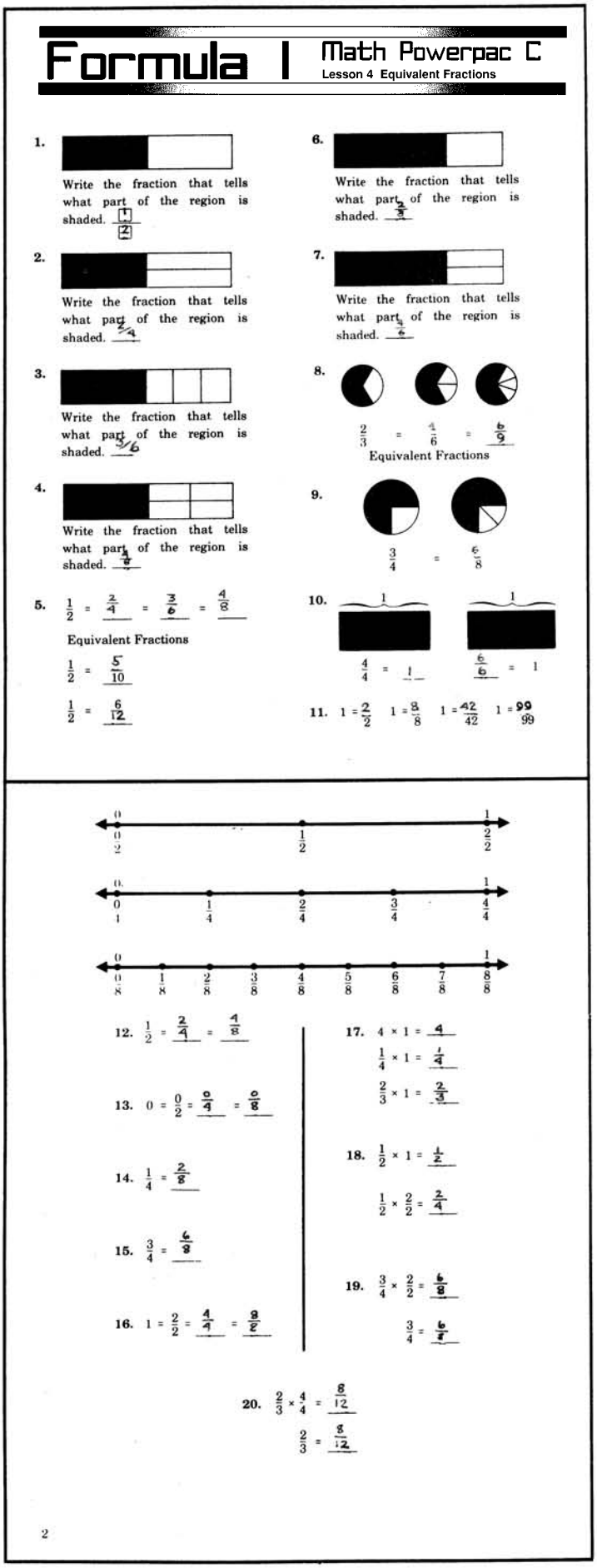
equivalent fractions: fractions which name the same number

Evaluation: The student's performance on page 4 of the *activity pages,* which he completes after the *audio* has finished playing, should indicate his ability to work with equivalent fractions.

A Step Further

The following activities will provide able students an opportunity to further explore the topics presented in this lesson.

1. "Fraction strings" and "fraction stars" can provide interesting and challenging practice in finding equivalent fractions. Simply set up part of a "string" or "star," as shown below, and have students fill in the missing numerators and denominators to make equivalent fractions. (You might challenge the student to make his "fraction string" as long as he can.)
   Able students might also design strings and stars for others to complete. However, answer keys should be kept for easy checking.



21. $\frac{4}{5} = \frac{16}{20}$

$\frac{4}{5} \times \frac{4}{4} = \frac{16}{20}$

22. $\frac{3}{8} = \frac{9}{24}$

$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$

23. $\frac{5}{9} = \frac{20}{36}$

$\frac{5}{9} \times \frac{4}{4} = \frac{20}{36}$

24. $\frac{3}{7} = \frac{9}{21}$

$\frac{3}{7} \times \frac{3}{3} = \frac{9}{21}$

25. $\frac{4}{11} = \frac{16}{44}$

$\frac{4}{11} \times \frac{4}{4} = \frac{16}{44}$

26. $\frac{2}{3} = \frac{8}{12}$

$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$

27. $\frac{3}{9} = \frac{3 \times 1}{3 \times 3} = \frac{3}{3} \times \frac{1}{3} = \frac{1}{3}$

28. $\frac{5}{20} = \frac{5 \times 1}{5 \times 4} = \frac{1}{4}$

29. $\frac{12}{18} = \frac{6 \times 2}{6 \times 3} = \frac{2}{3}$

30. $\frac{15}{27} = \frac{3 \times 5}{3 \times 9} = \frac{5}{9}$

31. $\frac{6}{24} = \frac{6 \times 1}{6 \times 4} = \frac{1}{4}$

32. $\frac{7}{21} = \frac{7 \times 1}{7 \times 3} = \frac{1}{3}$

33. $\frac{16}{20} = \frac{4 \times 4}{4 \times 5} = \frac{4}{5}$

34. $\frac{18}{24} = \frac{6 \times 3}{6 \times 4} = \frac{3}{4}$

35. $\frac{8}{12} = \frac{4 \times 2}{4 \times 3} = \frac{2}{3}$

3

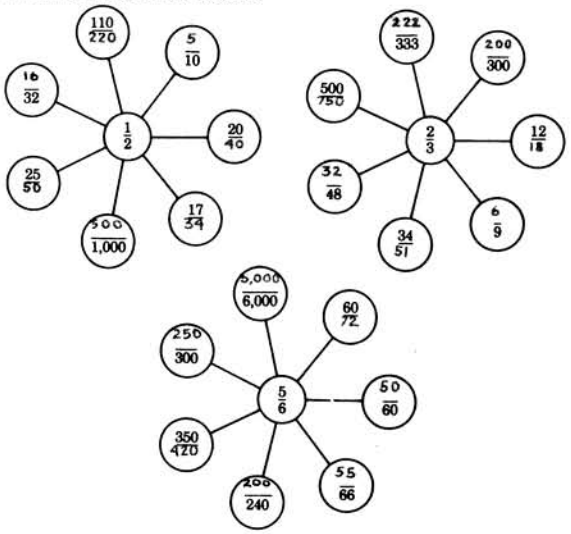
1. $\frac{1}{2} = \frac{2}{4} = \frac{10}{20} = \frac{9}{18}$
2. $\frac{5}{6} = \frac{15}{18}$

   $\frac{5}{6} \times \frac{3}{3} = \frac{15}{18}$
3. $0 = \frac{0}{4} = \frac{0}{8}$
4. $\frac{2}{9} \times \frac{3}{3} = \frac{2}{9}$
5. $\frac{4}{5} \times \frac{4}{4} = \frac{16}{20}$
6. $\frac{3}{4} \times \frac{9}{9} = \frac{27}{36}$
7. $\frac{1}{6} \times \frac{7}{7} = \frac{7}{42}$

Write in simplest form.

8. $\frac{45}{48} = \frac{15}{16}$
9. $\frac{14}{24} = \frac{7}{12}$
10. $\frac{17}{17} = 1$

Fill in the circles with equivalent fractions.

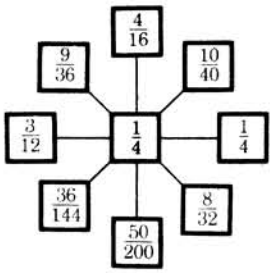


4

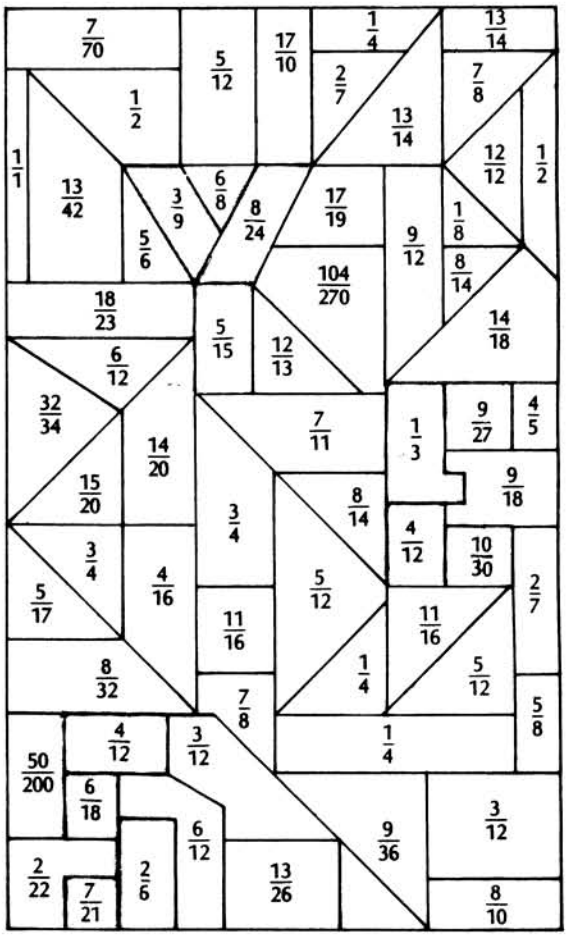
fraction string:

$\frac{4}{12} = \frac{12}{36} = \frac{6}{18} = \frac{1}{3} = \frac{5}{15} = \frac{17}{51}$

fraction star:



2. Instruct students to solve the puzzle below by shading each section that contains a fraction equivalent to $\frac{1}{3}$. If the correct parts are shaded, an immediate confirmation is revealed.



LESSON 5

Addition of Unlike Fractions

Approximate time required to complete the lesson: 40 min.

Background Needed

Before working in this lesson, the student should be able to add like fractions, find least common multiples, and understand prime factorization and equivalent fractions. If his skill in any of these areas seems questionable, it is suggested that you have him review selected earlier *audio* which teach those concepts.

In The Lesson

The need to rename unlike fractions to fractions with like denominators is demonstrated with number lines showing different fractional units. The lesson progresses to an explanation of simple shortcuts for finding the least common denominator (LCD) in problems with prime number denominators and in problems in which one denominator is a factor of the other. The use of multiples and prime factorization to find the LCD is also explained. Problems including both improper fractions and mixed numerals are presented.

Vocabulary: The following terms are introduced and used by the narrator during this lesson.

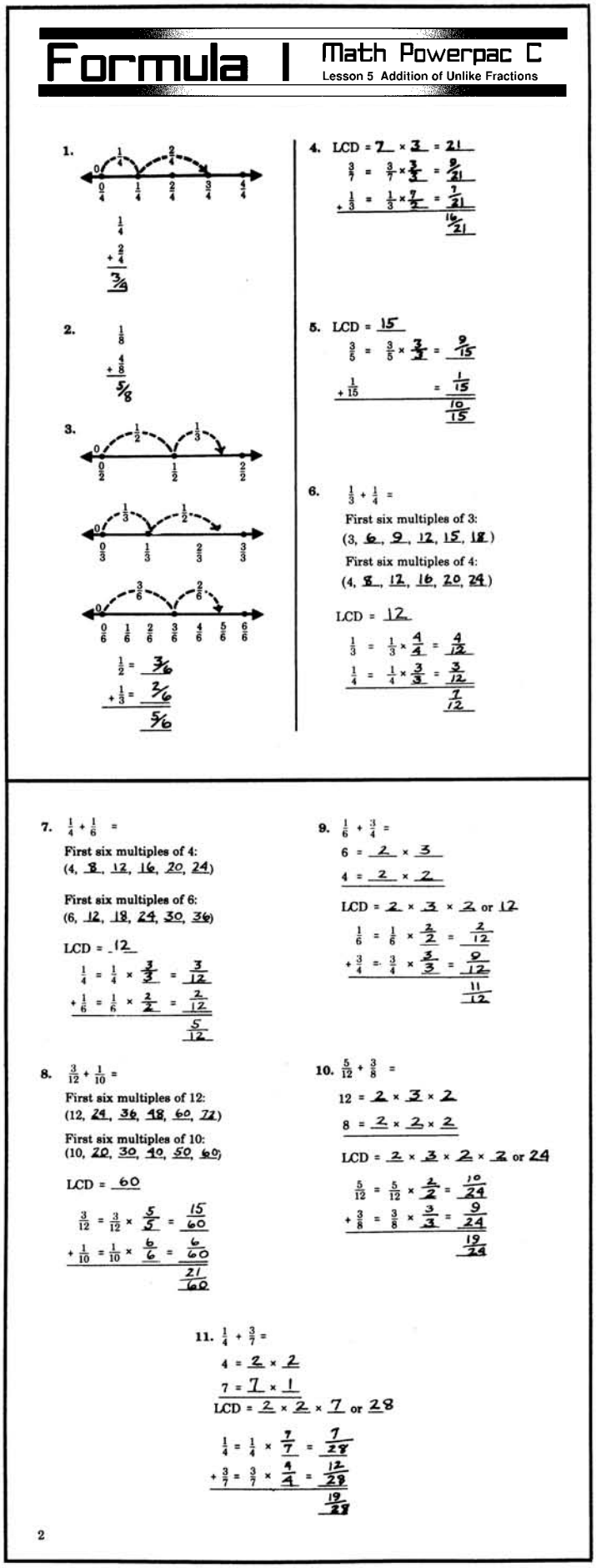
- **unlike fractions:** fractions whose denominators are not the same
- **least common denominator:** the least common multiple of two or more denominators
- **mixed numeral:** any numeral named as a whole number and a fractional number less than one
- **improper fraction:** a fraction whose numerator is the same size or larger than its denominator

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, will give you an indication of how well he understands least common denominatores and their application to addition of unlike fractions.

A Step Further

The following activities are suggested to provide able students with opportunities to expand their understanding of fractions.

1. Introduce square addition as an interesting method of practicing addition of fractions. If



12. $\frac{5}{9} = \frac{5}{9}$, $+\frac{1}{3} = \frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$, $\frac{8}{9}$

13. $\frac{2}{5} = \frac{4}{10}$, $+\frac{1}{2} = \frac{5}{10}$, $\frac{9}{10}$

14. $4\frac{4}{7}$, $+2\frac{1}{7}$, $6\frac{5}{7}$

15. $16\frac{3}{8}$, $+9\frac{4}{8}$, $25\frac{7}{8}$

16. $3\frac{1}{2} = 3\frac{2}{4}$, $+2\frac{1}{4} = 2\frac{1}{4}$, $5\frac{3}{4}$

17. $5\frac{2}{5} = 5\frac{4}{10}$, $+3\frac{3}{10} = 3\frac{3}{10}$, $8\frac{7}{10}$

18. $\frac{9}{8} = \frac{8}{8} + \frac{1}{8}$, $= 1\frac{1}{8}$

19. $\frac{13}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5}$, $= 2\frac{3}{5}$

20. $\frac{11}{8} = 1\frac{3}{8}$

21. $\frac{23}{10} = 2\frac{3}{10}$

22. $4\frac{3}{4} = 4\frac{6}{8}$, $+6\frac{5}{8} = 6\frac{5}{8}$, $10\frac{11}{8}$ or $11\frac{3}{8}$

23. $5\frac{2}{7} = 5\frac{22}{77}$, $+6\frac{3}{11} = 6\frac{21}{77}$, $11\frac{43}{77}$

3

1. $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$

2. $\frac{2}{9} + \frac{5}{9} = \frac{7}{9}$

3. $\frac{1}{5} + \frac{4}{5} = \frac{5}{5}$ or 1

4. $\frac{3}{4} = \frac{9}{12}$, $+\frac{2}{3} = \frac{8}{12}$, $\frac{17}{12}$ or $1\frac{5}{12}$

5. $\frac{5}{6} = \frac{20}{24}$, $+\frac{3}{8} = \frac{9}{24}$, $\frac{29}{24}$ or $1\frac{5}{24}$

6. $\frac{1}{6} = \frac{3}{18}$, $+\frac{5}{9} = \frac{10}{18}$, $\frac{13}{18}$

7. $\frac{2}{3} = \frac{6}{9}$, $+\frac{7}{9} = \frac{7}{9}$, $\frac{13}{9}$ or $1\frac{4}{9}$

8. $4\frac{5}{8}$, $+2\frac{2}{8}$, $6\frac{7}{8}$

9. $6\frac{7}{12} = 6\frac{7}{12}$, $+5\frac{3}{4} = 5\frac{9}{12}$, $11\frac{16}{12}$ or $12\frac{1}{3}$

10. $20\frac{5}{8} = 20\frac{15}{24}$, $+14\frac{1}{12} = 14\frac{2}{24}$, $34\frac{17}{24}$

11. $7\frac{5}{7} = 7\frac{15}{21}$, $+3\frac{2}{3} = 3\frac{14}{21}$, $10\frac{29}{21}$ or $11\frac{8}{21}$

12. $8\frac{2}{5} = 8\frac{4}{10}$, $+6\frac{7}{10} = 6\frac{7}{10}$, $14\frac{11}{10}$ or $15\frac{1}{10}$

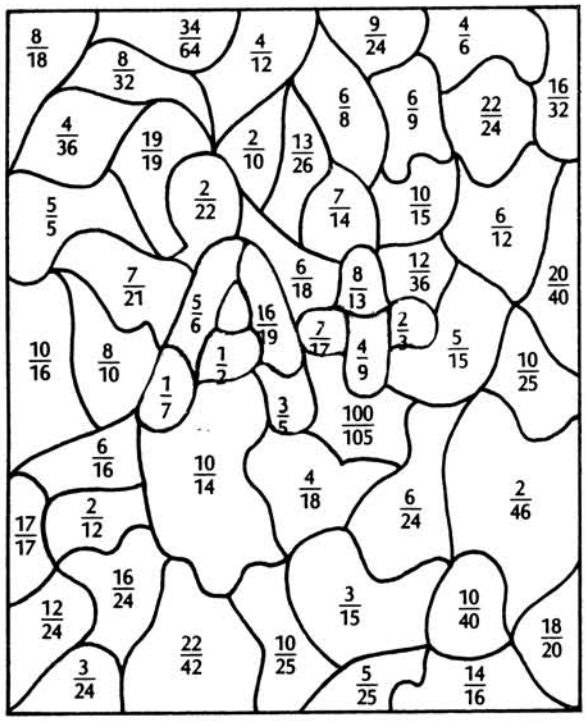
4

the missing addends in the internal squares are filled in correctly, the partial sums along the bottom and the right side will add up to the sum in the external square.

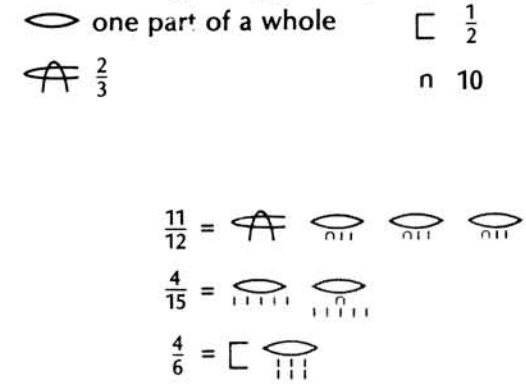
$\frac{1}{2}$	$\frac{1}{8}$	$= \frac{5}{8}$
$\frac{3}{8}$	$\frac{5}{8}$	$= \frac{8}{8}$
$\frac{7}{8}$	$\frac{6}{8}$	$\frac{13}{8}$

$\frac{1}{3}$	$\frac{3}{6}$	$= \frac{5}{6}$
$\frac{3}{12}$	$\frac{1}{6}$	$= \frac{5}{12}$
$\frac{7}{12}$	$\frac{4}{6}$	$\frac{15}{12}$

2. In the puzzle below, have students shade in the spaces containing the fractions which are stated in lowest terms. Explain that if they do all their shading correctly, the puzzle will "grade their work."



3. Since Egyptians had symbols only for unit fractions (fractions with a numeral of 1), and for the fractions $\frac{2}{3}$ and $\frac{1}{2}$, it was necessary to combine these symbols to write all other fractions. After you explain this system, ask students to write fractions using the Egyptian symbolism.



LESSON 6

Subtraction of Unlike Fractions

Approximate time required to complete the lesson: 30 min.

Background Needed

To benefit fully from this lesson, the student should be able to subtract fractions with like denominators without difficulty.

In The Lesson

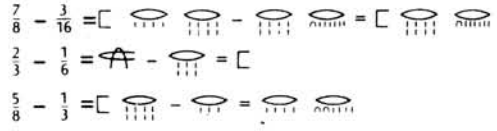
Following a brief review of subtracting fractions with like denominators, the narrator introduces the student to several shortcuts for finding the LCD. Prime factorization is included and is used extensively. The student is then guided to solve mixed numeral problems which require renaming both whole numbers and fractions.

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate his ability to use the subtraction skills taught in this lesson.

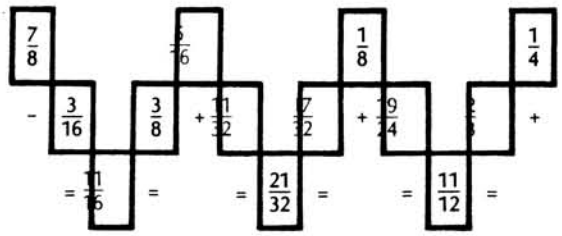
A Step Further

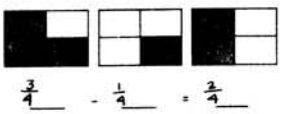
The following activities will provide able students an opportunity to expand their skill in subtracting unlike fractions.

1. Reintroduce the Egyptian fraction system and have students translate the problems below into Egyptian symbolism. (See page 11 for symbols.) Remind them that the Egyptians were limited to symbols for unit fractions, $\frac{2}{3}$, and $\frac{1}{2}$.

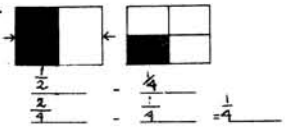


2. For challenging practice in renaming and subtracting fractions, let students complete the interlocking fraction chain below. You might also have them make up a chain of their own for other members of the class to solve.



1. 
$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

2. $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$

3. 
$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

4. $\frac{5}{7} - \frac{2}{3}$
LCD = 21
$\left(\frac{5}{7}\times\frac{3}{3}\right) - \left(\frac{2}{3}\times\frac{7}{7}\right)$
$\frac{15}{21} - \frac{14}{21} = \frac{1}{21}$

5. $\frac{3}{4} - \frac{5}{8}$
The first three multiples of 4 (4, 8, 12)
$\left(\frac{3}{4}\times\frac{2}{2}\right) - \frac{5}{8}$
$\frac{6}{8} - \frac{5}{8} = \frac{1}{8}$

6. $\frac{5}{6} - \frac{3}{4}$
$\frac{5}{6}\times\frac{2}{2} = \frac{10}{12}$
$\frac{3}{4}\times\frac{3}{3} = \frac{9}{12}$
$\frac{1}{12}$
Work Space: 6 3 ×2; 4 2 ×2; LCD 3 2 2; or 2

7. $\frac{11}{12} - \frac{5}{8} = \frac{11}{12}\times\frac{2}{2} - \frac{5}{8}\times\frac{3}{3} = \frac{22}{24} - \frac{15}{24}$
$\frac{7}{24}$
Work Space: 12 2 2 3; 8 2 2 2; LCD 2 2 3; 2; or 24

8. $\frac{6}{8} - \frac{4}{6} = \frac{6}{8}\times\frac{3}{3} - \frac{4}{6}\times\frac{4}{4} = \frac{18}{24} - \frac{16}{24}$
$\frac{2}{24}$
$\frac{1}{12}$
Work Space: 8 2 2 2; 6 3 2; LCD 2 2 2; 3; or 24

9. $4\frac{6}{7} - 2\frac{4}{7}$
$2\frac{2}{7}$

10. $4\frac{2}{3} = 4\frac{8}{12}$
$-2\frac{1}{4} = 2\frac{3}{12}$
$2\frac{5}{12}$
Work Space: $\frac{4}{4}\times\frac{2}{3} = \frac{8}{12}$; $\frac{1}{4}\times\frac{3}{3} = \frac{3}{12}$; LCD 12

11. $8\frac{3}{5} = 8\frac{9}{15}$
$-3\frac{1}{3} = 3\frac{5}{15}$
$5\frac{4}{15}$
Work Space: $\frac{5}{5}\times\frac{3}{5} = \frac{9}{15}$; $\frac{5}{5}\times\frac{1}{3} = \frac{5}{15}$; LCD = 15

12. $12\frac{5}{6} = 12\frac{20}{24}$
$-5\frac{3}{8} = 5\frac{9}{24}$
$7\frac{11}{24}$
Work Space: $\frac{4}{4}\times\frac{5}{6} = \frac{20}{24}$; $\frac{3}{3}\times\frac{3}{8} = \frac{9}{24}$; LCD 24

13. □□□□ = □□□⊟
$4 = 2\frac{2}{2}$

14. $5 = 4\frac{7}{7}$

15. $4\frac{\ }{\ } = 3\frac{8}{8}$

16. $7 = 6\frac{11}{11}$

17. $8 = 5\frac{5}{5}$

18. $10\frac{\ }{\ } = 9\frac{7}{7}$

2

19. $8 = 7\frac{5}{5}$
$-2\frac{3}{5} = 2\frac{3}{5}$
$5\frac{2}{5}$

20. $8\frac{7}{7}$
$-4\frac{6}{7}$
$4\frac{1}{7}$

21. $8\frac{4}{4}$
$7\frac{4}{4}$
$-3\frac{3}{4}$
$3\frac{1}{4}$

22. $\cancel{12}\ \frac{8}{8}$
$\cancel{11}\ 8$
$-4\frac{5}{8}$
$6\frac{3}{8}$

23. $\cancel{12}\ \frac{6}{6}$
$\cancel{10}\ 6$
$-9\frac{5}{6}$
$10\frac{1}{6}$

24. $\cancel{12}\ \frac{3}{3}$
$\cancel{9}\ 8$
$-8\frac{1}{3}$
$5\frac{2}{3}$

25. $4\frac{1}{4} = 3\frac{5}{4}$
$-2\frac{3}{4} = 2\frac{3}{4}$
$1\frac{2}{4}$ or $1\frac{1}{2}$

26. $6\frac{1}{5} = 5\frac{6}{5}$
$-2\frac{4}{5} = 2\frac{4}{5}$
$3\frac{2}{5}$

27. $8\frac{2}{3} = 8\frac{8}{12} = 7\frac{20}{12}$
$-2\frac{3}{4} = 2\frac{9}{12} = 2\frac{9}{12}$
$5\frac{11}{12}$

Work Space
$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$
$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$
LCD = 12

28. $5\frac{1}{8} = 5\frac{3}{24} = 4\frac{27}{24}$
$-1\frac{5}{12} = 1\frac{10}{24} = 1\frac{10}{24}$
$3\frac{17}{24}$

Work Space
$\frac{1}{8} \times \frac{3}{3} = \frac{3}{24}$
$\frac{5}{12} \times \frac{2}{2} = \frac{10}{24}$
LCD = 24

3

1. $\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$

2. $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$

3. $\frac{4}{5} = \frac{12}{15}$
$-\frac{2}{3} = \frac{10}{15}$
$\frac{2}{15}$

4. $\frac{7}{8} = \frac{7}{8}$
$-\frac{3}{4} = \frac{6}{8}$
$\frac{1}{8}$

5. $6 = 5\frac{4}{4}$

6. $6\frac{4}{4}$
$-2\frac{3}{4}$
$4\frac{1}{4}$

7. $9\frac{1}{5} = 8\frac{6}{5}$
$-2\frac{3}{5} = 2\frac{3}{5}$
$6\frac{3}{5}$

8. $8 = 7\frac{7}{7}$
$-3\frac{4}{7} = 3\frac{4}{7}$
$4\frac{3}{7}$

9. $14\frac{7}{9}$
$-5\frac{2}{9}$
$9\frac{5}{9}$

10. $20\frac{5}{6} = 20\frac{10}{12}$
$-7\frac{3}{4} = 7\frac{9}{12}$
$13\frac{1}{12}$

11. $18\frac{1}{3} = 18\frac{4}{12} = 17\frac{16}{12}$
$-10\frac{3}{4} = 10\frac{9}{12} = 10\frac{9}{12}$
$7\frac{7}{12}$

12. $7\frac{2}{5} = 7\frac{4}{10} = 6\frac{14}{10}$
$-2\frac{7}{10} = 2\frac{7}{10} = 2\frac{7}{10}$
$4\frac{7}{10}$

4

3. In 1530, Rudolff, a mathematician, introduced his method of adding fractions with unlike denominators. Your students may enjoy experimenting with this unique approach to addition of fractions. Without using the addition sign, write the addends between two horizontal lines. Below the bottom line, write the LCD and, above the top line, write the new numerator for the equivalent fractions with like denominators. Add the new numerators and record the sum to the right of the problem. You might give your students problems similar to the following:

$$\frac{\begin{matrix}8 & 9\\ \frac{2}{3} & \frac{3}{4}\end{matrix}}{12} = \frac{17}{12} \text{ or } 1\frac{5}{12}$$

Formula 1 Math Powerpac D

LESSON 1

Multiplication of Fractions

Approximate time required to complete the lesson: 30 min.

Background Needed

The student should know how to write fractions in simplest form before beginning this lesson.

In The Lesson

The lesson begins with an explanation of the fact that the product may be smaller than either of the factors when multiplying fractions less than one. The narrator then leads the student through a series of problems involving multiplication of unit fractions and whole numbers. The process of renaming fractions as mixed numerals, and the associative property, are then employed in multiplication of whole numbers and fractions. The commutative and distributive properties, the property of zero, and the use of common factors to simplify a problem are all explained. The student is given ample opportunity to practice multiplying fractions in 2- and 3-factor problems.

Vocabulary: The following term is introduced and used by the narrator in this lesson.

unit fraction: a fraction that has the numeral one as its numerator

Evaluation: **The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate his understanding of multiplication with fractions.**

A Step Further

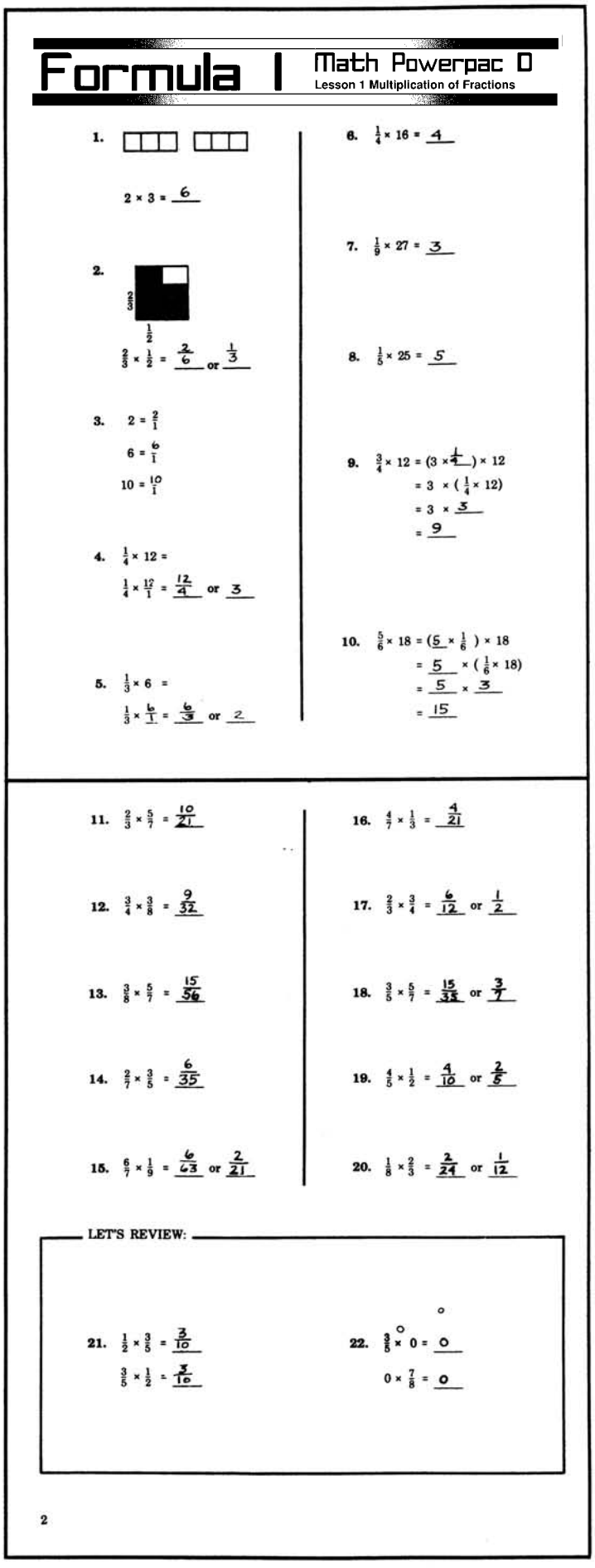
The following activities will help students expand their knowledge of multiplication of fractions.

1. A fraction sequence in which the members are determined by multiplication can provide interesting problem solving practice. Ask students to find the pattern for this sequence and then fill in the next three members.

   Example: $\frac{5}{8}, \frac{15}{16}, \frac{45}{32}, \frac{135}{64}, \frac{405}{128}, \frac{1215}{256}$

   Pattern: Multiply each member by $\frac{3}{2}$.

2. Students should enjoy solving a multiplication problem with the help of an illustration. Give each one a sheet of graph paper and tell them how to solve the problem, $\frac{1}{2} \times \frac{11}{16}$.



Formula 1 Math Powerpac D
Lesson 1 Multiplication of Fractions

1. $2 \times 3 = 6$

2. $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$ or $\frac{1}{3}$

3. $2 = \frac{2}{1}$, $6 = \frac{6}{1}$, $10 = \frac{10}{1}$

4. $\frac{1}{4} \times 12 = \frac{1}{4} \times \frac{12}{1} = \frac{12}{4}$ or 3

5. $\frac{1}{3} \times 6 = \frac{1}{3} \times \frac{6}{1} = \frac{6}{3}$ or 2

6. $\frac{1}{4} \times 16 = 4$

7. $\frac{1}{9} \times 27 = 3$

8. $\frac{1}{5} \times 25 = 5$

9. $\frac{3}{4} \times 12 = (3 \times \frac{1}{4}) \times 12$
   $= 3 \times (\frac{1}{4} \times 12)$
   $= 3 \times 3$
   $= 9$

10. $\frac{5}{6} \times 18 = (5 \times \frac{1}{6}) \times 18$
    $= 5 \times (\frac{1}{6} \times 18)$
    $= 5 \times 3$
    $= 15$

11. $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

12. $\frac{3}{4} \times \frac{3}{8} = \frac{9}{32}$

13. $\frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

14. $\frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$

15. $\frac{6}{7} \times \frac{1}{9} = \frac{6}{63}$ or $\frac{2}{21}$

16. $\frac{4}{7} \times \frac{1}{3} = \frac{4}{21}$

17. $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$ or $\frac{1}{2}$

18. $\frac{3}{5} \times \frac{5}{7} = \frac{15}{35}$ or $\frac{3}{7}$

19. $\frac{4}{5} \times \frac{1}{2} = \frac{4}{10}$ or $\frac{2}{5}$

20. $\frac{1}{8} \times \frac{2}{3} = \frac{2}{24}$ or $\frac{1}{12}$

LET'S REVIEW:

21. $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$
    $\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$

22. $\frac{3}{5} \times 0 = 0$
    $0 \times \frac{7}{8} = 0$

2

23. $\frac{3}{8} \times \frac{4}{6} = \frac{1}{4}$

24. $\frac{5}{9} \times \frac{3}{10} = \frac{1}{6}$

25. $\frac{3}{14} \times \frac{7}{9} = \frac{1}{6}$

26. $\frac{4}{5} \times \frac{15}{16} = \frac{3}{4}$

27. $\frac{2}{3} \times \frac{6}{8} \times \frac{4}{9} = \frac{2}{9}$

28. $\frac{3}{5} \times \frac{10}{12} \times \frac{8}{9} = \frac{4}{9}$

29. $\frac{3}{4} \times \frac{8}{12} \times \frac{2}{3} = \frac{1}{3}$

30. $\frac{3}{10} \times \frac{5}{7} \times \frac{2}{5} = \frac{3}{35}$

31. $3\frac{2}{7} = 3 + \frac{2}{7}$
$= \frac{21}{7} + \frac{2}{7} = \frac{23}{7}$

32. $8\frac{4}{5} = \frac{8}{1} + \frac{4}{5}$
$= \frac{40}{5} + \frac{4}{5} = \frac{44}{5}$

33. $11\frac{1}{2} = \frac{11}{1} + \frac{1}{2}$
$= \frac{22}{2} + \frac{1}{2} = \frac{23}{2}$

34. $\frac{2}{3} \times 5\frac{1}{3}$
$\frac{2}{3} \times \frac{16}{3} = \frac{32}{9}$ or $3\frac{5}{9}$

35. $\frac{7}{8} \times 3\frac{5}{6}$
$\frac{7}{8} \times \frac{23}{6} = \frac{161}{48}$ or $3\frac{17}{48}$

36. $\frac{4}{9} \times 6\frac{2}{3}$
$\frac{4}{9} \times \frac{20}{3} = \frac{80}{27}$ or $2\frac{26}{27}$

37. $\frac{1}{2} \times 2\frac{1}{3}$
$\frac{1}{2} \times (2 + \frac{1}{3})$
$(\frac{1}{2} \times 2) + (\frac{1}{2} \times \frac{1}{3})$
$1 + \frac{1}{6} = 1\frac{1}{6}$

38. $\frac{3}{4} \times 8\frac{1}{2}$
$\frac{3}{4} \times (8 + \frac{1}{2})$
$(\frac{3}{4} \times 8) + (\frac{3}{4} \times \frac{1}{2})$
$6 + \frac{3}{8} = 6\frac{3}{8}$

39. $\frac{5}{6} \times 12\frac{4}{9}$
$\frac{5}{6} \times (12 + \frac{4}{9})$
$(\frac{5}{6} \times 12) + (\frac{5}{6} \times \frac{4}{9})$
$10 + \frac{20}{54} = 10\frac{10}{27}$ or $10\frac{20}{54}$

3

Make sure all products are in *simplest form.*

1. $\frac{1}{4} \times 12 = 3$

2. $\frac{1}{8} \times 24 = 3$

3. $\frac{3}{5} \times 10 = 6$

4. $\frac{4}{7} \times 14 = 8$

5. $\frac{2}{3} \times \frac{7}{11} = \frac{14}{33}$

6. $\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$

7. $\frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$

8. $\frac{5}{8} \times \frac{16}{25} = \frac{2}{5}$

9. $\frac{2}{3} \times \frac{3}{5} \times \frac{5}{8} = \frac{1}{4}$

10. $\frac{6}{7} \times \frac{14}{3} \times \frac{4}{9} = \frac{16}{9}$ or $1\frac{7}{19}$

11. $(\frac{1}{2} \times \frac{2}{3}) \times \frac{3}{4} =$
$\frac{1}{2} \times (\frac{2}{3} \times \frac{3}{4}) =$
$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

12. $\frac{1}{4} \times 0 = 0$
$0 \times \frac{2}{3} = 0$

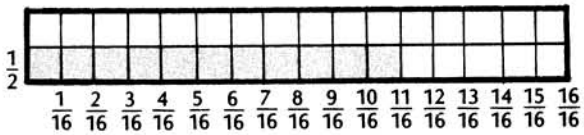
13. $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$
$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

14. $\frac{4}{7} \times 35 =$
$4 \times (\frac{1}{7} \times 35) =$
$4 \times 5 = 20$

15. $\frac{3}{8} \times 48 = 18$

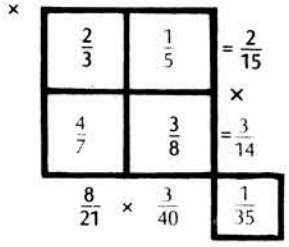
4

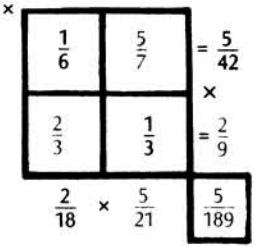
First, direct them to use rulers to draw two lines at right angles to each other. These represent the two denominators. One line should measure 2 units, for the denominator in $\frac{1}{2}$, and the other, 16 units, for the denominator in $\frac{11}{16}$.
Then draw two more lines to complete a box which will be 2 squares deep by 16 squares long. The entire box represents one whole which contains 32 units or squares.
Next, tell them to draw a horizontal line from the first factor, $\frac{1}{2}$, to intersect a line drawn vertically from the second factor, $\frac{11}{16}$. When they have made their lines, have them shade in the enclosed area. Call for the answer and the reason for the answer.



This box contains 11 squares, each of which is $\frac{1}{32}$, so the product of $\frac{1}{2} \times \frac{11}{16}$ is $\frac{11}{32}$.
Problem: $\frac{1}{2} \times \frac{11}{16} = \frac{11}{32}$

3. The square multiplication pattern provides fun and challenge in multiplication of fractions. Prepare squares similar to the following and ask students to fill in the missing factors and products. If all the work is done correctly, the products of the two problems outside the square will be identical when written in simplest form. This common product is then written in the small outside square.





LESSON 2

Division of Fractions

Approximate time required to complete the lesson: 35 min.

Background Needed

To participate successfully in this lesson, the student should understand division of whole numbers and multiplication of fractions. If his skill in either of these areas is questionable, it is suggested that he review selected *audio* dealing with those concepts.

In The Lesson

After a review of the identity element, the student is introduced to reciprocals. In several division problems involving whole numbers, the narrator demonstrates that the quotient remains the same when the divisor and dividend are multiplied by the same number. He then leads the student through a series of exercises involving division of fractions. Throughout the lesson, the pupil is directed to write the quotients in simplest form.

Vocabulary: The following terms are introduced and used by the narrator in this lesson.

reciprocals: a pair of numbers whose product is one; the fractional number obtained by interchanging the numerator and the denominator

property of one in division: any number (except zero) divided by itself is one

Evaluation: **The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate how well he understands division of fractions.**

A Step Further

The following activities will provide able students an opportunity to expand their understanding of division of fractions.

1. The square multiplication format may be adapted to provide valuable practice in division of fractions.

÷			
	$\frac{1}{6}$	$\frac{5}{7}$	$\frac{7}{30}$
	$\frac{2}{3}$	$\frac{1}{3}$	2
	$\frac{1}{4}$	$2\frac{1}{7}$	$\frac{7}{60}$

Formula 1 Math Powerpac D
Lesson 2 Divisions of Fractions

If 1 is a factor in a multiplication problem, the product of the problem will always be the same as the other factor.

1. $8 \times 1 = \boxed{8}$
2. $1 \times 20 = \boxed{20}$
3. $1 \times \boxed{7} = 7$

Any number other than zero divided by itself equals one.

4. $4 \div 4 = \boxed{1}$
5. $9 \div 9 = \boxed{1}$
6. $14 \div 14 = \boxed{1}$
7. $\frac{2}{3} \times \frac{3}{2} = \boxed{\frac{6}{6}}$ or $\boxed{1}$
8. $\frac{5}{4} \times \frac{4}{5} = \boxed{\frac{20}{20}}$ or $\boxed{1}$
9. $\frac{6}{1} \times \frac{1}{6} = \boxed{\frac{6}{6}}$ or $\boxed{1}$
10. $\frac{3}{4} \times \boxed{\frac{4}{3}} = 1$
11. $\boxed{\frac{7}{7}} \times \frac{7}{8} = 1$
12. $\boxed{\frac{5}{5}} \times \frac{6}{5} = 1$

When the product of two fractions is 1, each number is the reciprocal of the other.

13. $\frac{3}{7} \boxed{\frac{7}{3}}$
14. $\frac{3}{5} \boxed{\frac{5}{3}}$
15. $\frac{9}{8} \boxed{\frac{8}{9}}$

identity element, property of 1, reciprocal

16. $\frac{3}{7} \times \frac{7}{3} = 1$ reciprocal
17. $19 \div 19 = 1$ property of 1
18. $9 \times 1 = 9$ identity element

19. $8 \div 2 = \boxed{4}$

Dividend ÷ Divisor = Quotient

20. $(8 \times 2) \div (2 \times 2) =$

16 ÷ 4 = 4

21. $(8 \times 3) \div (2 \times 3) =$

24 ÷ 6 = 4

$(8 \times 5) \div (2 \times 5) =$

40 ÷ 10 = 4

$(8 \times 10) \div (2 \times 10) =$

80 ÷ 20 = 4

22. $\frac{2}{3} \div \frac{3}{4} =$

$(\frac{2}{3} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3})$

$(\frac{2}{3} \times \frac{4}{3}) \div 1$

$\frac{8}{9} \div 1 = \boxed{\frac{8}{9}}$

23. $\frac{1}{3} \div \frac{1}{2} =$

$(\frac{1}{3} \times \frac{2}{\square}) \div (\frac{1}{2} \times \frac{2}{\square})$

$\frac{2}{3} \div 1 = \frac{2}{3}$

24. $\frac{3}{8} \div \frac{5}{6} =$

$\frac{3}{8} \times \frac{6}{5} = \frac{18}{40}$ or $\frac{9}{20}$

25. $\frac{2}{3} \div \frac{5}{7} =$

$\frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$

26. $\frac{3}{4} \div \frac{7}{8} =$

$\frac{3}{4} \times \frac{8}{7} = \frac{24}{28}$ or $\frac{6}{7}$

27. $\frac{5}{6} \div \frac{3}{4} =$

$\frac{5}{6} \times \frac{4}{3} = \frac{20}{18}$ or $\frac{10}{9}$

2

28. $4 \div \frac{1}{3}$

$\frac{4}{1} \times \frac{3}{1} = 12$

29. $9 \div \frac{1}{3}$

$\frac{9}{1} \times \frac{3}{1} = 27$

30. $12 \div \frac{1}{2}$

$\frac{12}{1} \times \frac{2}{1} = 24$

31. $\frac{7}{8} \div \frac{3}{5}$

$\frac{7}{8} \times \frac{5}{3} = \frac{35}{24}$ or $1\frac{11}{24}$

32. $\frac{3}{5} \div \frac{2}{7}$

$\frac{3}{5} \times \frac{7}{2} = \frac{21}{10}$ or $2\frac{1}{10}$

33. $\frac{5}{6} \div \frac{5}{12}$

$\frac{5}{6} \times \frac{12}{5} = \frac{60}{30}$ or 2

Put in +, −, ×, or ÷ to make each sentence true. Use ( ) when necessary.

$(4 - 4) \div 4 = 0$ or $(4 - 4) \times 4 = 0$

$(4 \div 4) + 4 = 5$

$(4 \div 4) \div 4 = \frac{1}{4}$

3

1. What is the reciprocal of $\frac{7}{8}$? $\frac{8}{7}$

2. What is the reciprocal of $\frac{9}{5}$? $\frac{5}{9}$

3. $8 \times 1 =$ 8

4. $11 \div 11 =$ 1

5. $19 \div 19 =$ 1

After each example, write *identity element*, *property of one*, or *reciprocal*.

6. $5 \div 5 = 1$ one

7. $\frac{3}{5} \times 1 = \frac{3}{5}$ identity element

8. $\frac{3}{8} \times \frac{8}{3} = 1$ reciprocal

9. $\frac{1}{7} + \frac{3}{4} =$

$(\frac{1}{7} \times \frac{4}{3}) + (\frac{3}{4} \times \frac{4}{3})$

$\frac{4}{21} + 1 = \frac{4}{21}$

10. $2 \div \frac{3}{5} =$

$2 \times \frac{5}{3} = \frac{10}{3}$ or $3\frac{1}{3}$

11. $\frac{1}{2} \div \frac{5}{8} =$

$\frac{1}{2} \times \frac{8}{5} = \frac{8}{10}$ or $\frac{4}{5}$

12. $\frac{2}{3} \div \frac{5}{7} =$

$\frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$

13. $\frac{3}{10} \div \frac{2}{7} =$

$\frac{3}{10} \times \frac{7}{2} = \frac{21}{20}$ $1\frac{1}{20}$

14. $4 \div \frac{3}{5} =$

$\frac{4}{1} \times \frac{5}{3} = \frac{20}{3}$ or $6\frac{2}{3}$

15. $\frac{1}{3} \div \frac{5}{5} = \frac{5}{9}$

$\frac{1}{3} \times \frac{5}{5} = \frac{5}{9}$

4

+

$\frac{1}{35}$	$\frac{8}{21}$	$\frac{3}{40}$
$\frac{3}{14}$	$\frac{4}{7}$	$\frac{3}{8}$
$\frac{2}{15}$	$\frac{2}{3}$	$\frac{1}{5}$

2. The following table provides interesting practice in performing all four operations with fractions. Students must fill in the missing numerals in each vertical row. In the first row, for instance, $\frac{1}{4}$ and $\frac{2}{3}$ are added to get a sum of $\frac{11}{12}$. The same numbers, $\frac{1}{4}$ and $\frac{2}{3}$, are then multiplied to obtain a product of $\frac{1}{6}$. In the second column, they must first subtract to find what number plus $\frac{1}{3}$ equals $\frac{7}{12}$, and then multiply to find the product.

Sum	$\frac{11}{12}$	$\frac{7}{12}$	$1\frac{5}{12}$	$\frac{53}{56}$	$1\frac{4}{55}$	$1\frac{5}{12}$	$\frac{23}{24}$
	$\frac{1}{4}$	$\frac{1}{3}$	$1\frac{1}{4}$	$\frac{4}{7}$	$\frac{3}{11}$	$\frac{3}{4}$	$\frac{5}{8}$
	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{1}{3}$
Product	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{5}{24}$	$\frac{3}{14}$	$\frac{12}{55}$	$\frac{1}{2}$	$\frac{5}{24}$

3. Ask students to write the whole numbers from 1 to 10, using only one digit. To do so, they must rename each number and use one or more of the basic operations. If they need assistance, you might give them the pattern for one or two numbers.

The pattern:

$1 = \frac{44}{44}$ or $\frac{4}{4} \times \frac{4}{4}$ or $(\frac{4}{4} + 4) - 4$

$2 = \frac{4}{4} + \frac{4}{4}$ or $\frac{4 \times 4}{4 + 4}$

$3 = \frac{(4 + 4 + 4)}{4}$

$4 = 4 + \{4 \times (4 - 4)\}$

$5 = \frac{4 + (4 \times 4)}{4}$

$6 = 4 + \frac{(4 + 4)}{4}$

$7 = 4 + (4 - \frac{4}{4})$ or $\frac{44}{4} - 4$

$8 = 4 + 4 + 4 - 4$ or $\frac{(4 + 4)}{(4 \div 4)}$

$9 = 4 + 4 + \frac{4}{4}$

$10 = \frac{(44 - 4)}{4}$

(Any digit may be used in this pattern. Ingenious students may discover other patterns.)

LESSON 3

Mixed Fractions

Approximate time required to complete the lesson: 45 min.

Lesson 3 includes all the fraction operations presented in earlier lessons. Therefore, the student should have a working knowledge of the four basic operations with fractions before starting this lesson.

In The Lesson

This lesson begins with an introduction to the meaning of mixed numerals. A brief review of fractions is followed by a series of instructional steps carrying the student through activities involving all four operations with mixed fractions. The lesson provides practice in renaming whole numbers as fractions, renaming improper fractions as mixed numerals, and the use of reciprocals.

Special Instructions: The student should be provided with a sheet of scratch paper on which to do the computations needed to complete some of the responses. Page 3 of the *activity pages* is the review page, which the student is told to complete immediately after the *audio* has finished playing. Page 4 consists of challenging enrichment exercises which he is instructed to complete only if the teacher has told him to do so. It is important that you tell the student, before the *audio* begins, whether or not he is to complete the optional fourth page.

Vocabulary: The following term is introduced and used by the narrator in this lesson.

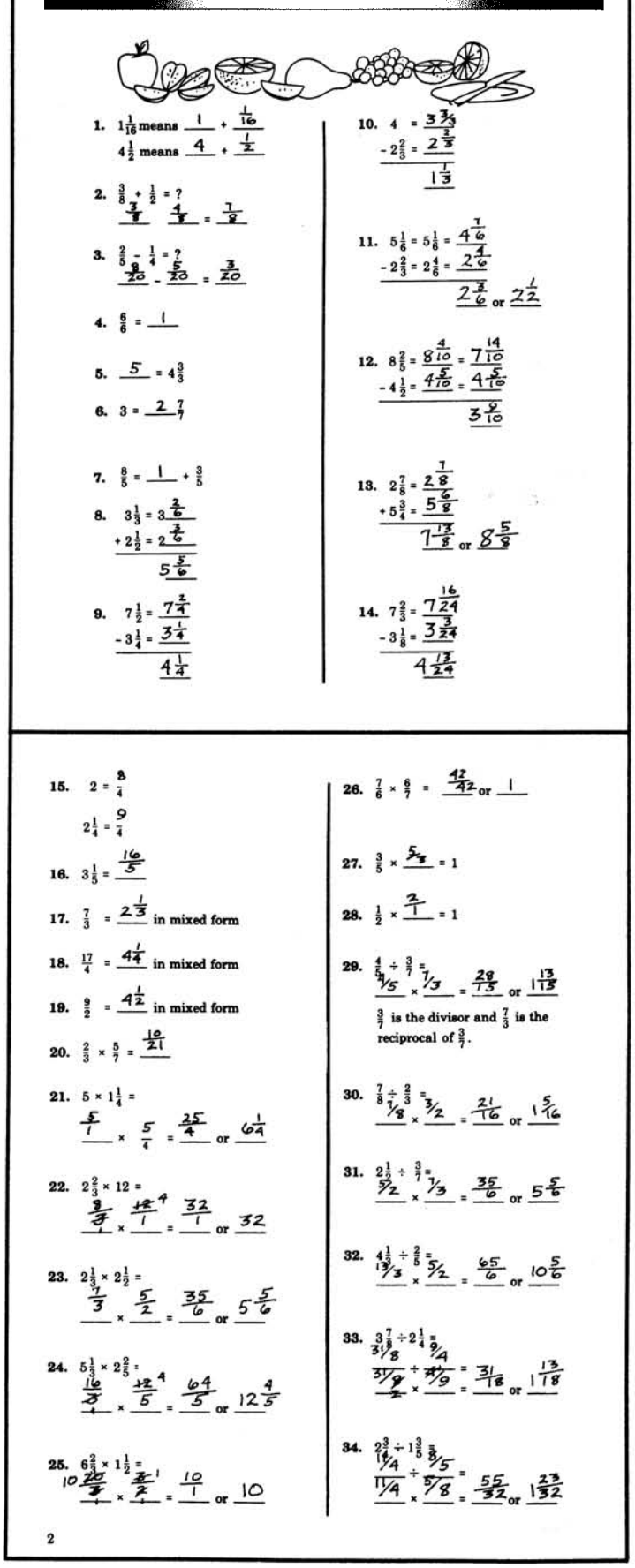
mixed numeral: a whole number and a fractional number less than one

Evaluation: The student's performance on page 3, which he completes after the *audio* has finished playing, should indicate his understanding of operation with mixed numberals.

A Step Further

The following activities are suggested to give able students an opportunity to expand their knowledge of operations with mixed numerals.

1. Ask students to fill in the "mischievous matrix" shown at the top of page 7, so that the sum of the numbers, when taken horizontally, vertically, or diagonally, is $3\frac{15}{16}$.



Formula 1 Math Powerpac D
Lesson 3 Mixed Fractions

1. $1\frac{1}{16}$ means $\frac{1}{1} + \frac{1}{16}$
   $4\frac{1}{2}$ means $\frac{4}{1} + \frac{1}{2}$
2. $\frac{3}{8} + \frac{1}{2} = ?$
   $\frac{3}{8} \quad \frac{4}{8} = \frac{7}{8}$
3. $\frac{2}{5} - \frac{1}{4} = ?$
   $\frac{8}{20} - \frac{5}{20} = \frac{3}{20}$
4. $\frac{6}{6} = \frac{1}{1}$
5. $\frac{5}{1} = 4\frac{3}{3}$
6. $3 = 2\frac{7}{7}$
7. $\frac{8}{5} = \frac{1}{1} + \frac{3}{5}$
8. $3\frac{1}{3} = 3\frac{2}{6}$
   $+ 2\frac{1}{2} = 2\frac{3}{6}$
   $5\frac{5}{6}$
9. $7\frac{2}{4} = 7\frac{2}{4}$
   $- 3\frac{1}{4} = 3\frac{1}{4}$
   $4\frac{1}{4}$
10. $4 = 3\frac{3}{3}$
    $- 2\frac{2}{3} = 2\frac{2}{3}$
    $1\frac{1}{3}$
11. $5\frac{1}{6} = 5\frac{1}{6} = 4\frac{7}{6}$
    $- 2\frac{2}{3} = 2\frac{4}{6} = 2\frac{4}{6}$
    $2\frac{3}{6}$ or $2\frac{1}{2}$
12. $8\frac{2}{5} = 8\frac{4}{10} = 7\frac{14}{10}$
    $- 4\frac{1}{2} = 4\frac{5}{10} = 4\frac{5}{10}$
    $3\frac{9}{10}$
13. $2\frac{7}{8} = 2\frac{7}{8}$
    $+ 5\frac{3}{4} = 5\frac{6}{8}$
    $7\frac{13}{8}$ or $8\frac{5}{8}$
14. $7\frac{2}{3} = 7\frac{16}{24}$
    $- 3\frac{1}{8} = 3\frac{3}{24}$
    $4\frac{13}{24}$

15. $2 = 2\frac{4}{4}$
    $2\frac{1}{4} = \frac{9}{4}$
16. $3\frac{1}{5} = \frac{16}{5}$
17. $\frac{7}{3} = 2\frac{1}{3}$ in mixed form
18. $\frac{17}{4} = 4\frac{1}{4}$ in mixed form
19. $\frac{9}{2} = 4\frac{1}{2}$ in mixed form
20. $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$
21. $5 \times 1\frac{1}{4} =$
    $\frac{5}{1} \times \frac{5}{4} = \frac{25}{4}$ or $6\frac{1}{4}$
22. $2\frac{2}{3} \times 12 =$
    $\frac{8}{3} \times \frac{12}{1} = \frac{32}{1}$ or 32
23. $2\frac{1}{3} \times 2\frac{1}{2} =$
    $\frac{7}{3} \times \frac{5}{2} = \frac{35}{6}$ or $5\frac{5}{6}$
24. $5\frac{1}{3} \times 2\frac{2}{5} =$
    $\frac{16}{3} \times \frac{12}{5} = \frac{64}{5}$ or $12\frac{4}{5}$
25. $6\frac{2}{3} \times 1\frac{1}{2} =$
    $\frac{20}{3} \times \frac{3}{2} = \frac{10}{1}$ or 10
26. $\frac{7}{6} \times \frac{6}{7} = \frac{42}{42}$ or 1
27. $\frac{3}{5} \times \frac{5}{3} = 1$
28. $\frac{1}{2} \times \frac{2}{1} = 1$
29. $\frac{4}{5} \div \frac{3}{7} =$
    $\frac{4}{5} \times \frac{7}{3} = \frac{28}{15}$ or $1\frac{13}{15}$
    $\frac{3}{7}$ is the divisor and $\frac{7}{3}$ is the reciprocal of $\frac{3}{7}$.
30. $\frac{7}{8} \div \frac{2}{3} =$
    $\frac{7}{8} \times \frac{3}{2} = \frac{21}{16}$ or $1\frac{5}{16}$
31. $2\frac{1}{2} \div \frac{3}{7} =$
    $\frac{5}{2} \times \frac{7}{3} = \frac{35}{6}$ or $5\frac{5}{6}$
32. $4\frac{1}{3} \div \frac{2}{5} =$
    $\frac{13}{3} \times \frac{5}{2} = \frac{65}{6}$ or $10\frac{5}{6}$
33. $3\frac{7}{8} \div 2\frac{1}{4} =$
    $\frac{31}{8} \div \frac{9}{4} = \frac{31}{18}$ or $1\frac{13}{18}$
34. $2\frac{3}{4} \div 1\frac{3}{5} =$
    $\frac{11}{4} \div \frac{8}{5} = \frac{55}{32}$ or $1\frac{23}{32}$

2

1. $4\frac{1}{2} = 4\frac{4}{8}$
$+2\frac{5}{8} = 2\frac{5}{8}$
$6\frac{9}{8}$ or $7\frac{1}{8}$

2. $6\frac{2}{3} = 6\frac{8}{12}$
$-2\frac{1}{4} = 2\frac{3}{12}$
$4\frac{5}{12}$

3. $5\frac{5}{5}$
$-2\frac{3}{5}$
$3\frac{2}{5}$

4. $4\frac{1}{2} \times 12 =$
$\frac{9}{2} \times \frac{12}{1} = 54$

5. $2\frac{2}{3} \times 3\frac{5}{8} =$
$\frac{8}{3} \times \frac{29}{8} = \frac{29}{3}$ or $9\frac{2}{3}$

6. $8\frac{1}{2} = 8\frac{4}{8} = 7\frac{12}{8}$
$-3\frac{7}{8} = 3\frac{7}{8} = 3\frac{7}{8}$
$4\frac{5}{8}$

7. $4\frac{1}{3} \div 2\frac{1}{2} =$
$\frac{13}{3} \div \frac{5}{2} = \frac{26}{15} = 1\frac{11}{15}$

8. $7\frac{1}{6} = 7\frac{9}{54}$
$+2\frac{5}{9} = 2\frac{30}{54}$
$9\frac{39}{54}$ or $9\frac{13}{18}$

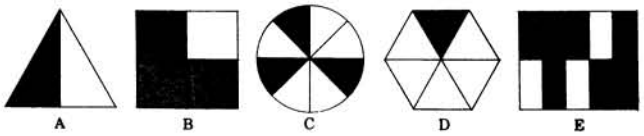
9. $3\frac{3}{4} \times 2\frac{1}{5} =$
$\frac{15}{4} \times \frac{11}{5} = \frac{33}{4} = 8\frac{1}{4}$

10. $2\frac{5}{6} \div 1\frac{1}{2} =$
$\frac{17}{6} \div \frac{3}{2} = \frac{17}{9} = 1\frac{8}{9}$

11. $8\frac{7}{7}$
$-2\frac{5}{7}$
$5\frac{2}{7}$

12. $8\frac{2}{3} = 8\frac{16}{24}$
$+5\frac{5}{8} = 5\frac{15}{24}$
$13\frac{31}{24} = 14\frac{7}{24}$

3



A B C D E

Use the figures above to help you fill in the correct numbers in the chart below. You will need to stop and think.

	Figure	Value of Shaded Part	Value of Unshaded Part	Total Value
Example	A	$2\frac{2}{3}$	$2\frac{2}{3}$	$5\frac{1}{3}$
	B	15	5	20
	C	$\frac{3}{7}$	$\frac{5}{7}$	$\frac{8}{7}$ or $1\frac{1}{7}$
	D	3	15	18
	E	28	12	40
	A	$3\frac{1}{4}$	$3\frac{1}{4}$	$6\frac{1}{2}$
	C	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{8}{4}$ or 2
	D	$\frac{3}{4}$	$3\frac{3}{4}$	$3\frac{6}{4}$ or $4\frac{1}{2}$
	E	$\frac{14}{3}$ or $4\frac{2}{3}$	$\frac{6}{3}$ or 2	$6\frac{2}{3}$
	A	$1\frac{7}{8}$	$1\frac{7}{8}$	$2\frac{14}{8}$ or $3\frac{3}{4}$
*	B	$\frac{21}{8}$ or $2\frac{5}{8}$	$\frac{7}{8}$	$3\frac{1}{2}$
*	C	$2\frac{1}{2}$	$\frac{40}{6}$ or $4\frac{4}{6}$	$\frac{40}{6}$ or $6\frac{2}{3}$

* Brain Twisters

4

$1\frac{1}{2}$	$1\frac{1}{16}$	$1\frac{3}{8}$
$1\frac{3}{16}$	$1\frac{5}{16}$	$1\frac{7}{16}$
$1\frac{1}{4}$	$1\frac{9}{16}$	$1\frac{1}{8}$

To construct additional matrices of this type, use any arithmetic progression of mixed numerals, arranged in the order shown in the matrix below. An arithmetic progression is any sequence in which each succeeding member is larger by a constant amount. The smallest member of the sequence goes in the box marked 1, the next in the box marked 2, etc. (For the sample given above, the sequence starts at $1\frac{1}{16}$ and increases by constant increments of $\frac{1}{16}$.)

8	1	6
3	5	7
4	9	2

After you have constructed the matrix, give the student a copy containing only three members. These must be any three not in the same row, column, or diagonal.

2. The square operations introduced in Lessons 1 and 2 can provide valuable practice in working with mixed numerals. The following examples may be varied according to your student's needs or abilities.

×

$5\frac{1}{3} \times \frac{14}{7}$		$10\frac{2}{3}$
$\frac{8}{7} \times 1\frac{4}{5}$		$\times\ 2\frac{2}{35}$
$6\frac{2}{21} \times 3\frac{21}{35}$		$21\frac{33}{35}$

+

$2\frac{5}{6} + \frac{8}{9}$		$= 3\frac{13}{18}$
$2\frac{1}{3} + 2\frac{1}{4}$		$= 4\frac{7}{12}$
$5\frac{1}{6} + 3\frac{5}{36}$		$8\frac{11}{36}$

÷

$10\frac{1}{2}$	$3\frac{1}{9}$	$3\frac{3}{8}$
$1\frac{3}{4}$	$2\frac{1}{3}$	$\frac{3}{4}$
6	$1\frac{1}{3}$	$4\frac{1}{2}$

−

$9\frac{2}{3}$	$2\frac{5}{12}$	$7\frac{3}{12}$
$6\frac{5}{12}$	$1\frac{4}{12}$	$5\frac{1}{12}$
$3\frac{3}{12}$	$1\frac{1}{12}$	$2\frac{2}{12}$

LESSON 4

Addition and Subtraction of Decimals

Approximate time required to complete the lesson: 40 min.

Background Needed

To perform successfully in this lesson, the student should know the two basic operations of addition and subtraction and understand place value.

In The Lesson

After a series of exercises that teach rounding off to the nearest tenth, hundredth, and thousandth, a number line is introduced to show equivalent decimal fractions. The student is guided to discover the importance of the decimal point in a series of addition and subtraction problems in which the digits remain constant, but the decimal point is moved. The use of decimals to solve word problems is also included in the lesson.

Special Instructions: The student will need a sheet of scratch paper on which to complete the computations for the problems in this lesson.

Vocabulary: The following terms are introduced and used by the narrator in this lesson.

round off: to simplify a decimal number by approximating it to the nearest tenth, hundredth, etc.

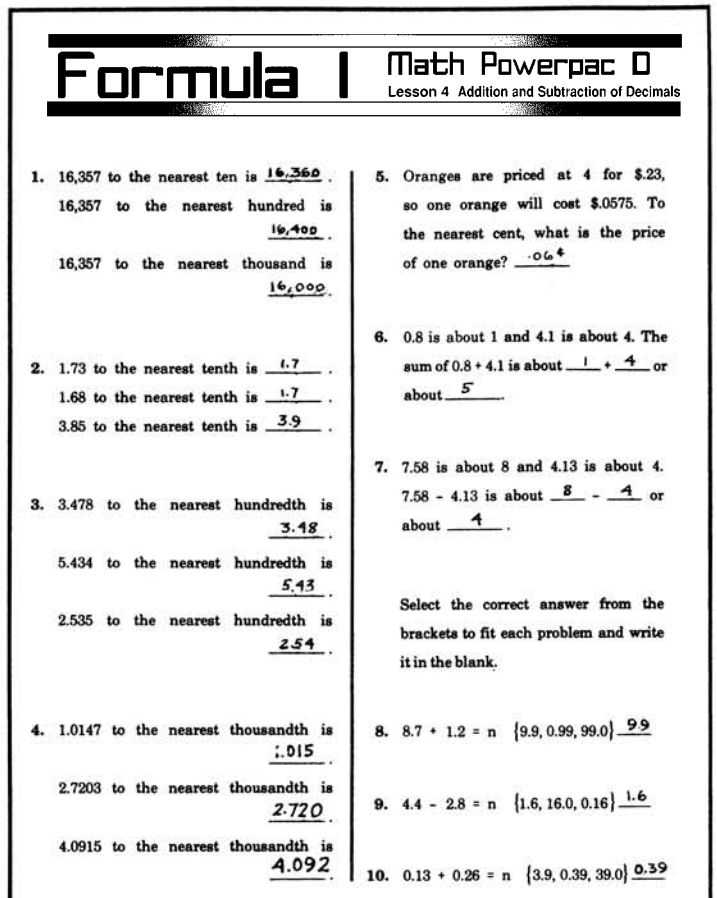
perimeter: the distance around a polygon

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should give you an indication of how well he understands the addition and subtraction of decimals.

A Step Further

The following activities are suggested to give students an opportunity to expand their understanding of operations with decimals.

1. Ask students to estimate the answer for each of these problems to the nearest whole number. Then ask them to solve the problem to find the actual sum.
   a. 4.35 + 0.89 + 2.795 = 8 (estimate) 8.035 (actual sum)
   b. 18.93 – 10.67 = 8 (estimate) 8.26 (actual answer)
   c. 6.45 + 0.689 + 23.69 = 31 (estimate) 30.829 (actual sum)



Formula 1 | Math Powerpac D
Lesson 4 Addition and Subtraction of Decimals

1. 16,357 to the nearest ten is 16,360.
   16,357 to the nearest hundred is 16,400.
   16,357 to the nearest thousand is 16,000.

2. 1.73 to the nearest tenth is 1.7.
   1.68 to the nearest tenth is 1.7.
   3.85 to the nearest tenth is 3.9.

3. 3.478 to the nearest hundredth is 3.48.
   5.434 to the nearest hundredth is 5.43.
   2.535 to the nearest hundredth is 2.54.

4. 1.0147 to the nearest thousandth is 1.015.
   2.7203 to the nearest thousandth is 2.720.
   4.0915 to the nearest thousandth is 4.092.

5. Oranges are priced at 4 for $.23, so one orange will cost $.0575. To the nearest cent, what is the price of one orange? .06

6. 0.8 is about 1 and 4.1 is about 4. The sum of 0.8 + 4.1 is about 1 + 4 or about 5.

7. 7.58 is about 8 and 4.13 is about 4. 7.58 - 4.13 is about 8 - 4 or about 4.

Select the correct answer from the brackets to fit each problem and write it in the blank.

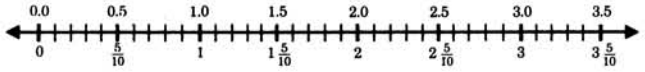
8. 8.7 + 1.2 = n {9.9, 0.99, 99.0} 9.9

9. 4.4 - 2.8 = n {1.6, 16.0, 0.16} 1.6

10. 0.13 + 0.26 = n {3.9, 0.39, 39.0} 0.39

11. 68 - 35 = 33

12. 6.8 - 3.5 = 3.3



13. 5/10 + 2/10 = 7/10

14. 0.5 + 0.2 = 0.7

15. 0.35 + 0.22 = 0.57

16. 3.86 - 1.45 = 2.41

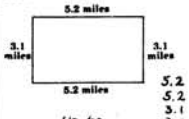
17. 0.644 + 0.254 = 0.898

18. 9.483 - 1.203 = 8.280

19. Add 423 + 1,907 + 609
    423 + 1907 + 609 = 2939

20. Add 2.341 + 4.426 + .002
    2.341 + 4.426 + .002 = 6.769

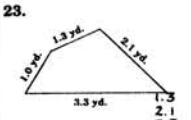
Find the distance around each figure.

21. 

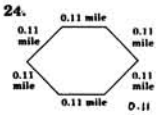
P = 16.6 miles (5.2 + 5.2 + 3.1 + 3.1 = 16.6)

22. 

P = 22.7 cm. (8.3 + 8.3 + 6.1 = 22.7)

23. 

P = 7.7 yd. (1.3 + 2.1 + 3.3 + 1.0 = 7.7)

24. 

P = 0.66 mile (0.11 + 0.11 + 0.11 + 0.11 + 0.11 + 0.11 = 0.66)

5.3 + 8.2 + 13.5 = 27.0

25. Henry rode his bicycle 5.3 miles to York, 8.2 miles to Greensville, and 13.5 miles back home. How many miles did Henry ride in all? 27 miles

2

26. 764 + 189 = 953

27. 7.64 + 1.89 = 9.53

28. 0.764 + 0.189 = 0.953

29. 851 − 267 = 584

30. 85.1 − 26.7 = 58.4

31. 8.51 − 2.67 = 5.84

32. 0.21 − 0.14 = 0.07

33. 5.36 + 4.54 = 9.90

34. 2.586 + 3.705 = 6.291

35. 10.39 − 2.61 = 7.78

36. 8.981 − 2.784 = 6.197

37. 0.266 + 0.417 + 1.347 = 2.030

38. 0.315 − 0.029 = n

0.286

39. 2.707 + 3.606 = n

2.707
3.606
6.313

40. The wing span of the X-15, the last of several supersonic research planes, is 22.36 ft. The X-1, the first supersonic research plane, had a wing span that was 6.05 feet greater than the X-15's. What was the wing span of the X-1? 28.41 ft.

22.36
+ 6.05
28.41

41. Caribou, Maine had 22.2 inches of snow in January and 7.6 inches in April. How much more snow fell in January than in April? 14.6 in.

22.2
− 7.6
14.6

3

1. 0.70 + 0.19 = 0.89

2. 2.07 + 1.38 = 3.45

3. 4.37 + 7.68 = 12.05

4. 4.06 + 7.18 + 2.29 = 13.53

5. 0.17 + 2.08 + 3.62 = 5.87

6. 0.09 + 4.37 + 9.86 = 14.32

7. 8.79 − 5.27 = 3.52

8. 12.67 − 9.23 = 3.44

9. 25.83 − 17.21 = 8.62

10. 4.84 − 2.57 = 2.27

11. 8.91 − 5.28 = 3.63

12. 6.03 − 2.81 = 3.22

13. 7.00 − 2.89 = 4.11

14. 8.23 − 4.69 = 3.54

15. 14.04 − 5.87 = 8.17

16. 3.720 + 8.917 = n

3.720
8.917
12.637 = n

17. 12.069 − 9.873 = n

12.069
9.873
2.196 = n

18. Mrs. Smith bought three steaks. One weighed 2.41 pounds, one weighed 1.86 pounds, and the third weighed 3.07 pounds. How many pounds of steak did she buy?

2.41
1.86
3.07
7.34

7.34 lbs.

4

2. You might ask several of your students to give a report on Simon Stevin, the first mathematician to concern himself with decimals. Stevin, born in 1548, spent much of his life in the Netherlands. In his first book, he did not use the term "decimal point," as we know it today. Instead, he called the whole number a "unit" and gave it the symbol ⓪; the tenth (of an integer) was called a "prime" and was given the symbol ①. The hundredth (of an integer) was called a "second" and given the symbol ②, and so on.

   Have the students who present the reports demonstrate Stevin's idea by applying it to some multiplication problems, as in the example below:

```
   ⓪①②
   4 3 9 5
     2 9 2
   ---------
     8 7 9 0
   3 9 5 5
   8 7 9 0
 -----------
   ⓪①②③④
 1 2 8 3 3 4 0
```

3. Further challenge for able students is offered in number sentences similar to those below. Tell students to find the missing numbers and fill in the boxes.

6.300 + [2.590] = 8.890

[7.7609] + 12.67 = 20.4309

[4.0283] + .0017 = 4.0300

[5.4510] + 2.68 = 8.1310

[11.5132] − 3.4519 = 8.0613

12.67 − [4.779] = 7.891

4. Have your students start in the middle, at the top and fill in the empty blocks in this "Decimal Block" puzzle. Numbers written in a top block are repeated in the blocks immediately below them and are used in the next problem.

   a = addend, s = sum

						a	a	s						
				s	a	3.14	.14	3.28	a	a				
		a	a	9.03	5.89	3.14		3.28	1.35	1.93	s			
	s	a	8.364	.666	9.03				1.35	1.93	3.28	a	a	
77.75	68.72	8.364	.666								3.28	.77	2.51	

LESSON 5

Multiplication of Decimals

Approximate time required to complete the lesson: 30 min.

Background Needed

To benefit fully from this lesson, the student must be able to multiply whole numbers and fractions and have an understanding of place value.

In The Lesson

The narrator introduces multiplication of decimal numbers by relating them to equivalent fractions in analogous problems. The student learns to determine the number of decimal places in the factors to find the number of places in the product. He is given practice in finding products with up to four decimal places.

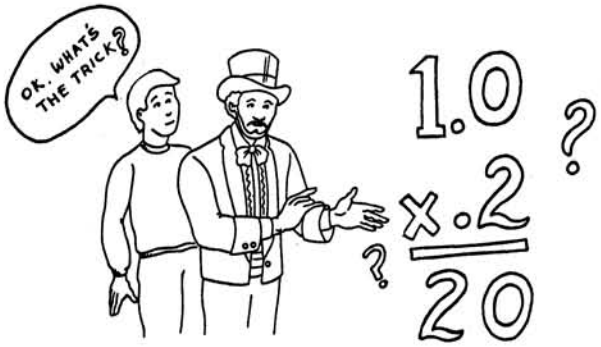
Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate how well he has learned multiplication of decimals from the *audio* lesson.

A Step Further

The following activities will provide able students an opportunity to explore multiplication of decimals in greater depth.

1. The two middle numerals in each column of this chart are addends which give a sum found in the top square. They are also factors which yield a product found in the bottom square. Have students use addition, subtraction, or multiplication to find each missing numeral.

Sums	.67	3.2	32.4	1.3	.58
	.07	.8	32	.7	.50
	.6	2.4	.4	.6	.08
Products	.042	1.92	12.8	.42	.0400



Formula 1 Math Powerpac D
Lesson 5 Multiplication of Decimals

1. $4 \times .1 = .4$; $4 \times \frac{1}{10} = \frac{4}{10}$

2. $3 \times .2 = .6$; $3 \times \frac{2}{10} = \frac{6}{10}$

3. $\begin{array}{r} 2 \\ \times .2 \\ \hline .4 \end{array}$

4. $\begin{array}{r} 2 \\ \times .3 \\ \hline .6 \end{array}$

5. $7 \times .3 = 2.1$; $7 \times \frac{3}{10} = \frac{21}{10}$ or $2\frac{1}{10}$

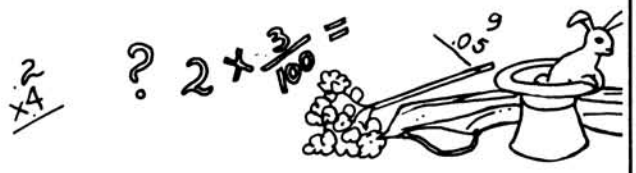
6. $8 \times .9 = 7.2$; $8 \times \frac{9}{10} = \frac{72}{10}$ or $7\frac{2}{10}$

7. $7 \times .7 = 4.9$

8. $\begin{array}{r} 5 \\ \times .5 \\ \hline 25 \end{array}$

9. $\begin{array}{r} 10 \\ \times .9 \\ \hline 90 \end{array}$

10. $\begin{array}{r} 10 \\ \times .6 \\ \hline 6.0 \end{array}$



11. $.3 \times .2 = .06$; $\frac{3}{10} \times \frac{2}{10} = \frac{6}{100}$

12. $.2 \times .2 = .04$; $\frac{2}{10} \times \frac{2}{10} = \frac{4}{100}$

13. $\begin{array}{r} .2 \\ \times .4 \\ \hline .08 \end{array}$

14. $\begin{array}{r} .3 \\ \times .3 \\ \hline .09 \end{array}$

15. $.7 \times .4 = .28$; $\frac{7}{10} \times \frac{4}{10} = \frac{28}{100}$

16. $\begin{array}{r} .8 \\ \times .7 \\ \hline .56 \end{array}$

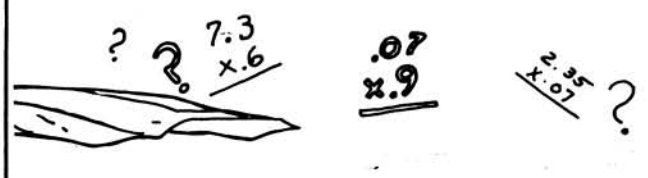
17. $\begin{array}{r} .5 \\ \times .3 \\ \hline .15 \end{array}$

18. $.9 \times 1.1 = .99$

19. $\begin{array}{r} 1.4 \\ \times .9 \\ \hline 1.26 \end{array}$

20. $\begin{array}{r} 7.3 \\ \times .6 \\ \hline 4.38 \end{array}$

2



21. $2 \times .03 =$ .06
$2 \times \frac{3}{100} = \frac{6}{100}$

22. 9
× .05
.45

23. 6
× .08
.48

24. $.9 \times .04 =$ .036
$\frac{9}{10} \times \frac{4}{100} = \frac{36}{1000}$

25. $.4 \times .08 =$ .032

26. .6
× .06
.036

27. .07
× .9
.063

28. $.04 \times .04 =$ .0016
$\frac{4}{100} \times \frac{4}{100} = \frac{16}{10000}$

29. .12
× .07
.0084

30. .09
× .05
.0045

31. 2.35
× .06
.1410

32. 8.19
× .06
.4914

3

1. $7 \times \frac{1}{100} = \frac{7}{100}$

2. $\frac{1}{1000} = .001$

3. How many decimal places are there in 7.009? 3

4. $.6 \times .3 = \frac{6}{10} \times \frac{3}{10} = \frac{18}{100} = 0.18$

5. $.8 \times .02 = \frac{8}{10} \times \frac{2}{100} = \frac{16}{1000} = 0.016$

6. 3.7
× 4
14.8

7. 4.05
× 00.3
1.215

8. 0.008
× 0.4
.0032

9. 3.8
× 4.6
228
152
17.48

10. 14.2
× 0.08
1.136

11. 1.46
× 3.7
1022
438
5402

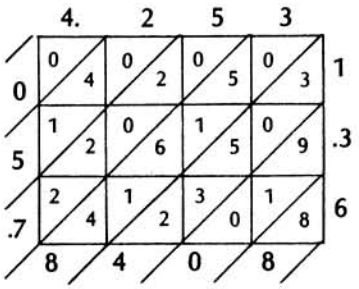
12. 42.07
× 0.19
37863
4207
7.9933

13. 4.809
× 0.07
.33663

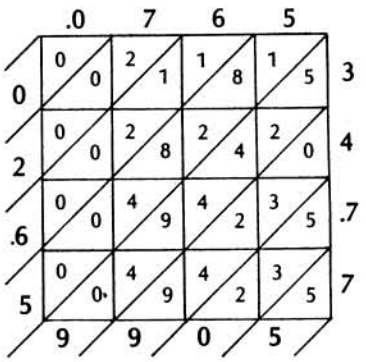
14. 407
× 0.006
2.442

4

2. The lattice multiplication method works well for decimal fractions. The decimal point is placed in the product just as it is in standard multiplication. Here are some examples:



Answer: 5.78408



Answer: 2.659905

3. Ask able students to circle the numeral that is an estimation of each product. You might also have them find the exact products and number the problems in sequence, beginning with the greatest product and continuing to the smallest.

Examples:

1. $12.0 \times 12 = ?$ (144.0, .1440, 14.40, 1.440)
2. $2.8 \times 2.8 = ?$ (6, 8.84, 18)
3. $8.0 \times .999 = ?$ (.7992, 7.992, 79.92, 799.2)
4. $35.6 \times .07 = ?$ (2.4, 24, 240, .24)
5. $4.5 \times .2 = ?$ (9, .09, .90, 900)
6. $0.7 \times .1 = ?$ (7, .7, .07, .007)

LESSON 6

Division of Decimals

Approximate time required to complete the lesson: 40 min.

Background Needed

To perform successfully in this lesson, the student should understand place value and be able to divide both fractions and whole numbers.

In The Lesson

The division of decimals is related to the student's previous experience with the division of fractions. Renaming the divisor as a whole number, the use of the identity element for multiplication, and multiplying the divisor and the dividend by the same power of ten are all explained. The student is guided in dividing tenths, hundredths, and thousandths.

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate how well he has understood the material presented on the *audio*.

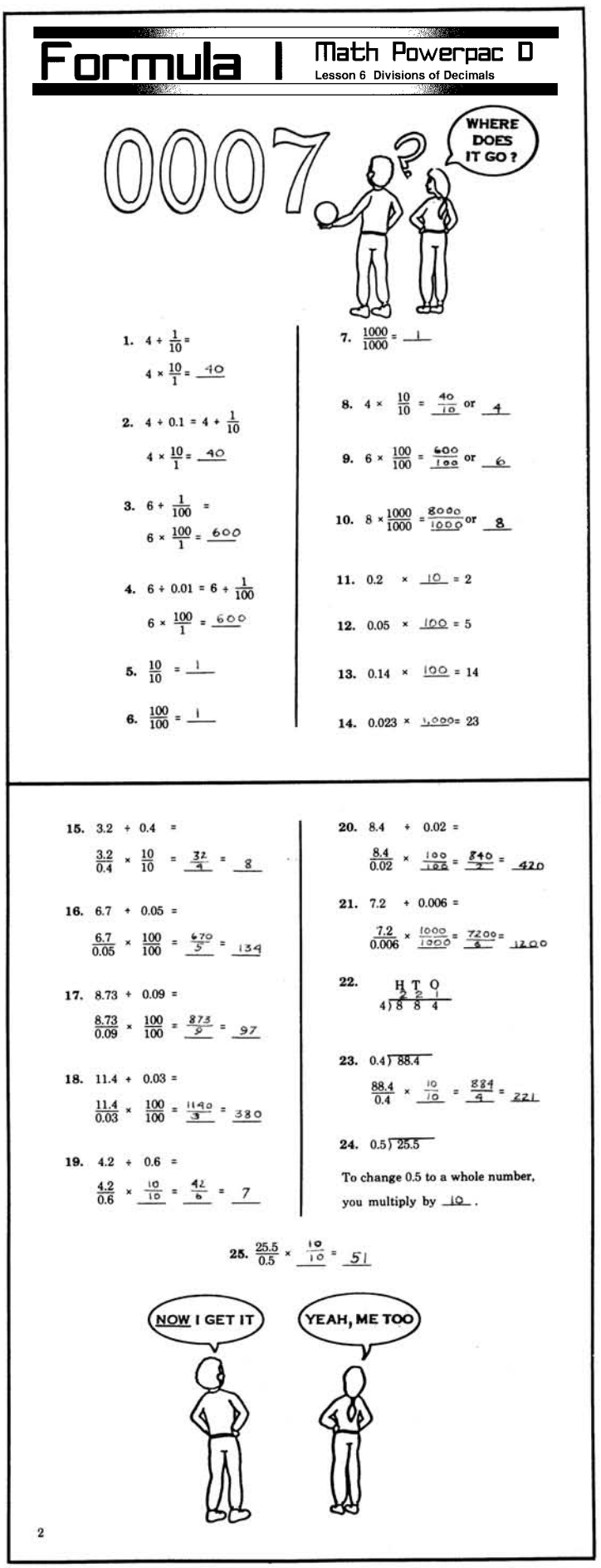
A Step Further

The following activities will provide able students an opportunity to explore further the topics presented in this lesson.

1. Introduce missing number and decimal group problems like those below. The difficulty of the division should be adjusted to the computation abilities of your students.

   .07 × $\frac{23.3}{}$ = 1.631
   .09 ÷ $\frac{.6}{}$ = .15
   .992 × $\frac{12.5}{}$ = 12.4
   $\frac{6393}{}$ × .002 = 12.786
   .17 × $\frac{6.3}{}$ = 1.071
   $\frac{6.35}{}$ ÷ .02 = 317.5

2. Distribute copies of the puzzle at the top of page 13 and have students fill in the signs for the correct operations.



Formula 1 Math Powerpac D
Lesson 6 Divisions of Decimals

0007 ?

1. $4 \div \frac{1}{10} =$
   $4 \times \frac{10}{1} =$ 40
2. $4 \div 0.1 = 4 \div \frac{1}{10}$
   $4 \times \frac{10}{1} =$ 40
3. $6 \div \frac{1}{100} =$
   $6 \times \frac{100}{1} =$ 600
4. $6 \div 0.01 = 6 \div \frac{1}{100}$
   $6 \times \frac{100}{1} =$ 600
5. $\frac{10}{10} =$ 1
6. $\frac{100}{100} =$ 1
7. $\frac{1000}{1000} =$ 1
8. $4 \times \frac{10}{10} = \frac{40}{10}$ or 4
9. $6 \times \frac{100}{100} = \frac{600}{100}$ or 6
10. $8 \times \frac{1000}{1000} = \frac{8000}{1000}$ or 8
11. 0.2 × 10 = 2
12. 0.05 × 100 = 5
13. 0.14 × 100 = 14
14. 0.023 × 1,000 = 23
15. 3.2 ÷ 0.4 =
    $\frac{3.2}{0.4} \times \frac{10}{10} = \frac{32}{4} =$ 8
16. 6.7 ÷ 0.05 =
    $\frac{6.7}{0.05} \times \frac{100}{100} = \frac{670}{5} =$ 134
17. 8.73 ÷ 0.09 =
    $\frac{8.73}{0.09} \times \frac{100}{100} = \frac{873}{9} =$ 97
18. 11.4 ÷ 0.03 =
    $\frac{11.4}{0.03} \times \frac{100}{100} = \frac{1140}{3} =$ 380
19. 4.2 ÷ 0.6 =
    $\frac{4.2}{0.6} \times \frac{10}{10} = \frac{42}{6} =$ 7
20. 8.4 ÷ 0.02 =
    $\frac{8.4}{0.02} \times \frac{100}{100} = \frac{840}{2} =$ 420
21. 7.2 ÷ 0.006 =
    $\frac{7.2}{0.006} \times \frac{1000}{1000} = \frac{7200}{6} =$ 1200
22. $4\overline{)8\ 8\ 4}$ H T O: 2 2 1
23. $0.4\overline{)88.4}$
    $\frac{88.4}{0.4} \times \frac{10}{10} = \frac{884}{4} =$ 221
24. $0.5\overline{)25.5}$
    To change 0.5 to a whole number, you multiply by 10.
25. $\frac{25.5}{0.5} \times \frac{10}{10} =$ 51

2

26.

$0.3\overline{)3.96}$

To change the divisor, 0.3, to a whole number, multiply by 10.

$0.3 \times 10 \overline{)3.96 \times 10}$

$$\begin{array}{r} 132 \\ 3\overline{)39\ 6} \\ 3 \\ \hline 09 \\ 9 \\ \hline 06 \\ 6 \\ \hline 0 \end{array}$$

27. $0.04\overline{)24.80}$

$0.04 \times 100 \overline{)24.80 \times 100}$

$$\begin{array}{r} 620 \\ 4\overline{)248\ 0} \\ 24 \\ \hline 08 \\ 8 \\ \hline 00 \\ 0 \end{array}$$

28. $0.9\overline{)0.216}$

$0.9 \times 10 \overline{)0.216 \times 10}$

$$\begin{array}{r} 0.24 \\ 9\overline{)2.16} \\ 18 \\ \hline 36 \\ 36 \\ \hline 0 \end{array}$$

29.

$$\begin{array}{r} 6.3 \\ 5\overline{)31.5} \\ 30 \\ \hline 15 \\ 15 \\ \hline 0 \end{array}$$

30.

$$\begin{array}{r} 440 \\ 8\overline{)3520} \\ 32 \\ \hline 32 \\ 32 \\ \hline 00 \\ 0 \end{array}$$

31.

$$\begin{array}{r} 21600 \\ 3\overline{)64800} \\ 6 \\ \hline 04 \\ 3 \\ \hline 018 \\ 18 \\ \hline 0 \end{array}$$

32.

$$\begin{array}{r} 0.36 \\ 7\overline{)0.252} \\ 21 \\ \hline 42 \\ 42 \\ \hline \end{array}$$

1. $\frac{4.8}{0.8} \times \frac{10}{10} = \frac{48}{8} = 6$

2. $\frac{7.3}{0.05} \times \frac{100}{100} = \frac{730}{5} = 146$

3. $0.03 \times 100 = 3$

4. $0.7 \times 10 = 7$

5. $0.017 \times 1000 = 17$

6. $\frac{4.08}{0.05} \times \frac{100}{100} = \frac{408}{5}$

7. $\frac{7.23}{0.9} \times \frac{10}{10} = \frac{72.3}{9}$

8. $\frac{23.4}{0.008} \times \frac{1000}{1000} = \frac{23400}{8}$

9. $0.3\overline{)4.2}$

$0.3 \times 10 \overline{)4.2 \times 10}$

10. $0.5\overline{)0.25}$

11. $0.08\overline{)25.60}$

12. $0.07\overline{)0.0049}$

13. $0.004\overline{)6.800}$

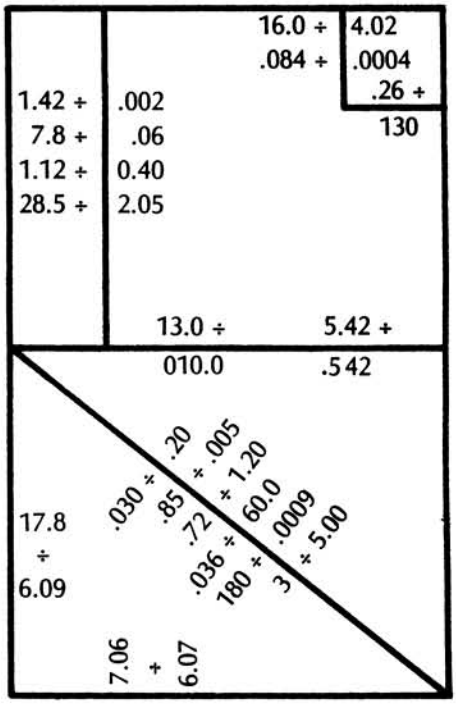
14. $0.9\overline{)3.42}$

15. $0.06\overline{)0.0288}$

16. $0.005\overline{)1.950}$

(.50	+	0.7)	÷	.3	=	4.0
+		×		×		×
(0.4	×	.40)	−	.07	=	.09
−		÷		+		+
(.007	−	.007)	+	.09	=	.09
=		=		=		=
.893		40		.111		4

3. Follow the example below in making a puzzle for each student. Students must first fit the puzzle pieces together to form a rectangle and then solve the decimal division problems formed by the pieces. The answers to the division sentences in the example are shown below.



1.42 ÷ .002 = 710
7.8 ÷ .06 = 130
1.12 ÷ 0.40 = 2.8
28.5 ÷ 2.05 = 13.9
16.0 ÷ 4.02 = 3.98
.084 ÷ .0004 = 210
.26 ÷ 130 = .002
13.0 ÷ 10.0 = 1.3

5.42 ÷ .542 = 10
.030 ÷ .20 = .15
.85 ÷ .005 = 170
.72 ÷ 1.20 = .6
.036 ÷ 60.0 = .0006
180 ÷ .0009 = 200,000
3 ÷ 5.00 = .6
17.8 ÷ 6.09 = 2.92
7.06 ÷ 6.07 = 1.16

Formula 1

Math Powerpac E

LESSON 1

Problems Involving Fractions and Decimals

Approximate time required to complete the lesson: 40 min..

Background Needed

To perform successfully in this lesson, the student should possess a working knowledge of the fundamentals of adding, subtracting, multiplying, and dividing with fractions and decimals.

In The Lesson

The narrator guides the student through the following four steps in solving problems involving fraction and decimals: deciding which operation to use, writing the number sentence, performing the computation, and labeling the answer. The *activity pages* provides avariety of exercises in which the student can use his newly gained knowledge.

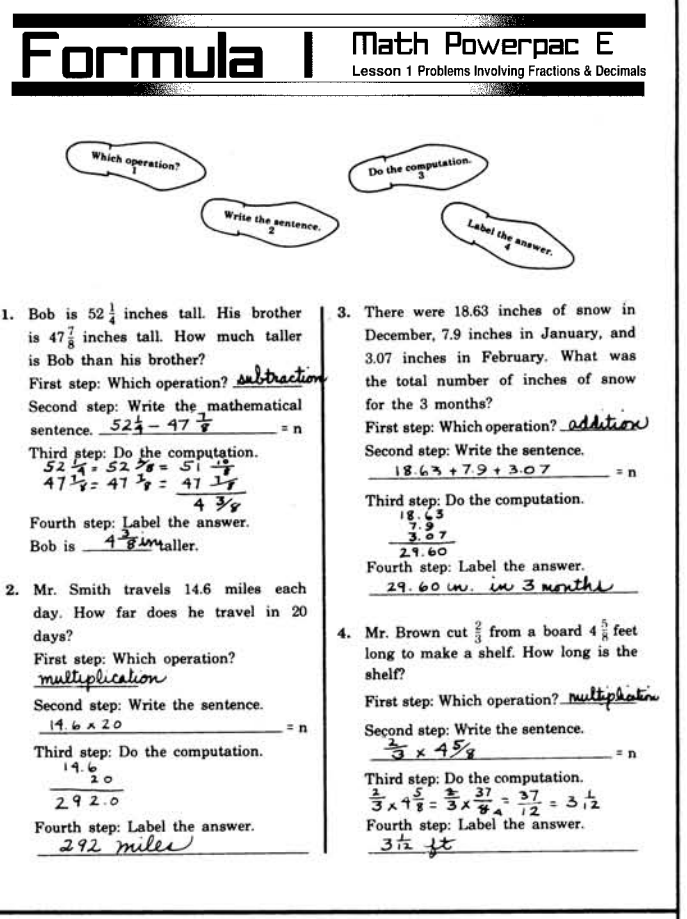
Special Instructions: The student will need a separate sheet of paper on which to perform computations for some of the problems on the *activity pages.*

Evaluation: The student's performance on page 4 of the *activity pages,* which he completes after the *audio* has finished playing, should indicate his ability to solve word problems involving fractions and decimals.

A Step Further

The following activities will provide able students an opportunity to practice using fractions and decimals in problem solving.

1. The history of the development of the "equals" symbol, a part of any mathematical equation, is an interesting one. You might ask interested students to report on Robert Recorde (1510-1585), the English mathematician who invented the symbol. You might also have them find out what was used before the "equals" sign was devised and demonstrate the older method on the board. (Words and abbreviations were used instead of symbols.)
2. Challenge your class with the case of endless fractions. Start with the following sequence: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$
   What are the next three members ? $\frac{1}{32}, \frac{1}{64}, \frac{1}{128}$



Formula | Math Powerpac E
Lesson 1 Problems Involving Fractions & Decimals

1. Bob is $52\frac{1}{4}$ inches tall. His brother is $47\frac{7}{8}$ inches tall. How much taller is Bob than his brother?
   First step: Which operation? subtraction
   Second step: Write the mathematical sentence. $52\frac{1}{4} - 47\frac{7}{8}$ = n
   Third step: Do the computation.
   $52\frac{1}{4} = 52\frac{2}{8} = 51\frac{10}{8}$
   $47\frac{7}{8} = 47\frac{7}{8} = 47\frac{7}{8}$
   $4\frac{3}{8}$
   Fourth step: Label the answer.
   Bob is $4\frac{3}{8}$ in. taller.

2. Mr. Smith travels 14.6 miles each day. How far does he travel in 20 days?
   First step: Which operation? multiplication
   Second step: Write the sentence. 14.6 x 20 = n
   Third step: Do the computation.
   14.6
   20
   292.0
   Fourth step: Label the answer.
   292 miles

3. There were 18.63 inches of snow in December, 7.9 inches in January, and 3.07 inches in February. What was the total number of inches of snow for the 3 months?
   First step: Which operation? addition
   Second step: Write the sentence. 18.63 + 7.9 + 3.07 = n
   Third step: Do the computation.
   18.63
   7.9
   3.07
   29.60
   Fourth step: Label the answer.
   29.60 in. in 3 months

4. Mr. Brown cut $\frac{2}{3}$ from a board $4\frac{5}{8}$ feet long to make a shelf. How long is the shelf?
   First step: Which operation? multiplication
   Second step: Write the sentence. $\frac{2}{3} \times 4\frac{5}{8}$ = n
   Third step: Do the computation.
   $\frac{2}{3} \times 4\frac{5}{8} = \frac{2}{3} \times \frac{37}{8_4} = \frac{37}{12} = 3\frac{1}{12}$
   Fourth step: Label the answer.
   $3\frac{1}{12}$ ft

5. Bob read that $\frac{2}{3}$ of a person's weight is water. He weighs 93 pounds. How much of Bob's weight is water?
   Multiplication
   $93 \times \frac{2}{3} = n$ $\quad \frac{^{31}93}{1} \times \frac{2}{3_1} = 62$ lbs.

6. Jim found a board $6\frac{1}{2}$ feet long. He cut off two pieces for a school project. One piece was $2\frac{1}{2}$ feet long and one was $1\frac{3}{4}$ feet long. How long was the piece of board left over?
   Addition $\quad 2\frac{1}{2} + 1\frac{3}{4} = 4\frac{1}{4}$
   Subtraction $\quad 6\frac{1}{2} - 4\frac{1}{4} = 2\frac{1}{4}$ ft. left over

7. The Girl's Club decided to make jumping ropes. How many $5\frac{1}{3}$-foot jumping ropes can they make from 80 feet of rope?
   Division
   $80 \div 5\frac{1}{3} = n$ $\quad \frac{^{5}80}{1} \times \frac{3}{16_1} = 15$ jumping ropes

8. The world land speed record was 148.63 miles per hour in 1936. The 1965 record was 600.601 miles per hour. How much faster was the 1965 record than the 1936 record?
   Subtraction
   $600.601 - 148.63 = n$
   600.601
   -148.63
   451.971 m.p.h. faster

2

9. A grocer is selling one brand of coffee at 75¢ per pound and another brand at 50¢ per pound. He wants to mix the two brands into 60 pounds of a blend that will sell for 65¢ per pound. How many pounds of 75¢ coffee and how many pounds of 50¢ coffee will he use?

a. Each pound of 75¢ coffee sold at 65¢ is a loss of 10 ¢.
b. Each pound of 50¢ coffee sold at 65¢ is a gain of 15 ¢.
c. 3 pounds of 75¢ coffee sold at 65¢ is a loss of 30 ¢.
d. 2 pounds of 50¢ coffee sold at 65¢ is a gain of 30 ¢.

The information above can be used to find the number of pounds of 75¢ coffee and the number of pounds of 50¢ coffee the grocer will use in making 60 pounds of blend. 60 ÷ 5 = 12 pkgs.

3 × 12 = 36 pounds of 75¢ coffee
2 × 12 = 24 pounds of 50¢ coffee
60 lbs. coffee in all

10. Tom can mow the lawn in 2 hours. Joe can mow the same lawn in 3 hours. How long will it take Tom and Joe to mow the lawn together?

a. How much of the lawn does Tom mow in 1 hour? $\frac{1}{2}$
b. How much of the lawn does Joe mow in 1 hour? $\frac{1}{3}$
c. How much of the lawn is mowed in 1 hour when Tom and Joe work together? $\frac{5}{6}$ $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

d. How much of the lawn remains to be mowed? $\frac{1}{6}$
e. How long will it take them to mow the lawn together? 1 hr. 12 min

60 min. = $\frac{5}{6}$ of lawn mowed
60 ÷ 5 = 12 min. to mow $\frac{1}{6}$ lawn
12 × 6 = 72 min. or 1 hr. 12 min. to mow entire lawn.

3

Write a mathematical sentence for each problem and solve the problem; then write the answer in a complete sentence.

1. During one week in the month of July, the following amounts of rainfall were recorded: 3.2 inches, 0.07 inches, and 0.3 inches. How much rain fell that week?

Addition
3.2
0.07
0.3
3.57 inches of rainfall in one week

2. A pound is 0.4539 kilograms. How many kilograms are there in 3 pounds?

Multiplication
1 lb. = 0.4539 kilograms
3 lbs. = 3 × 0.4539 or
1.3617 kilograms

0.4539
× 3
1.3617

3. George bought an 18-inch piece of balsa wood for his model ships. He cut off one piece $6\frac{3}{8}$ inches long and another $5\frac{3}{4}$ inches long. How much balsa wood did George have left?

Addition
$6\frac{3}{8} = 6\frac{3}{8}$
$+ 5\frac{3}{4} = 5\frac{6}{8}$
$11\frac{9}{8}$ or $12\frac{1}{8}$ in.

Subtraction
$18 = 17\frac{8}{8}$
$- 12\frac{1}{8} = 12\frac{1}{8}$
$5\frac{7}{8}$ in. left.

4. A group of Girl Scouts bought $4\frac{1}{2}$ yards of ribbon to make badges for field day. How many badges could they make if each badge were $\frac{1}{6}$ of a yard in length? Division $4\frac{1}{2} \div \frac{1}{6} = \frac{9}{2} \times \frac{6}{1} = 27$ badges

4

What is the sum of the first two members?
First three? First four? First five?
First six? First seven?
Could this process be continued until we have a sum equal to 1?
No. As the number of members increases, the sum approaches 1, but 1 can never be reached.

3. Ask students to write a mathematical sentence for each of the following problems and find the solution.

Sam, who lived in Arizona, collected pieces of turquoise for a friend in Illinois. During one week, he found turquoise pieces that weighed 32.0 grams, 7.7 grams, and 17.04 grams. If he mailed them to Illinois in a box that weighed 25.0 grams empty, how much did the whole package weigh?

32.0 + 7.7 + 17.04 + 25.0 = n

32.0
7.7
17.04
25.0
81.74 grams n = 81.74 grams

If one astronaut eats 1.4 pounds of food per day while in orbit, how much food would be needed by a three-man crew during a mission six days long?

6(3 × 1.4) = n
6(4.2) = n
n = 25.2 pounds of food

George, who was building a model airplane, bought an 18-inch piece of balsa wood. He needed a $6\frac{3}{8}$-inch piece for the body and a $5\frac{3}{4}$-inch piece for the wings. His sister Ann wanted a 6-inch piece of the wood for her dollhouse.
Was there enough wood left for Ann?

$18 - (6\frac{3}{8} + 5\frac{3}{4}) = n$
$18 - (6\frac{3}{8} + 5\frac{6}{8}) = n$
$18 - (12\frac{1}{8}) = n$
$n = 5\frac{7}{8}$ inches
There was not enough wood left for Ann.

LESSON 2

Ratio and Proportion

Approximate time required to complete the lesson: 25 min.

Background Needed

An understanding of *Equivalent Fractions* (Powerpac C, Lesson 4) will help the student gain full understanding of the material presented in this lesson.

In The Lesson

Ratio is presented in this lesson as a method of comparing two numbers. First, the student practices writing ratios in fractional form. Then, an explanation of equivalent ratios leads to a series of exercises involving proportions. The student is taught to use cross-multiplication as a check for proportions.

Vocabulary: The following terms and symbols are introduced and used by the narrator in this lesson.

ratio: a comparison of two numbers

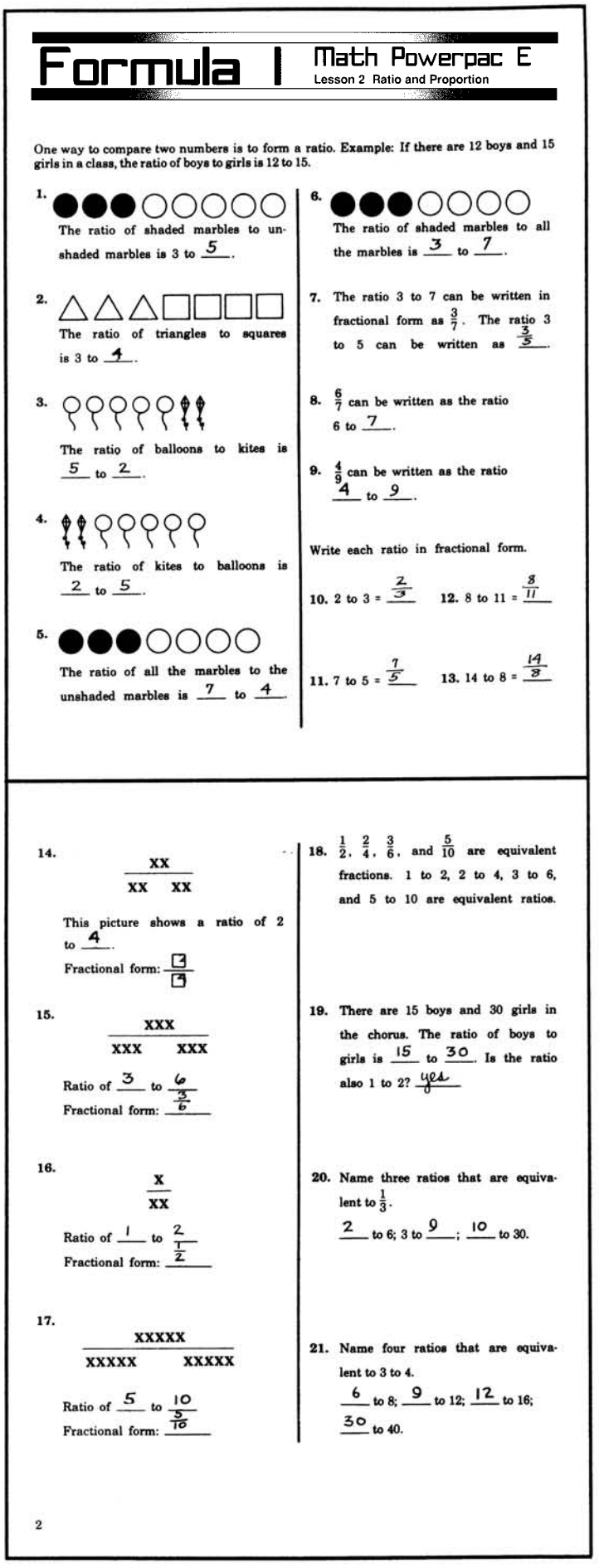
proportion: an equation that states the equality of two ratios

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, will indicate his understanding of ratio and proportion.

A Step Further

The following activities will give able students an opportunity to explore the concepts of ratio and proportion in greater depth.

1. To play "Rapid Ratio," divide a 9-inch circle into twelve equal sectors and write a ratio in each part. With a small paper fastener, fasten an arrow to the circle to serve as a spinner. For each player, mark off a 3- by 5-inch index card into nine equal squares. In each square, write a fraction that is equivalent to one of the ratios in the circle. Make sure the placement of the fractions on each card is different.
   Spin the arrow and have students cross out or cover the fraction on their card which is equivalent to the designated ratio. The first player to cross out or cover three correct fractions in



Formula 1 Math Powerpac E
Lesson 2 Ratio and Proportion

One way to compare two numbers is to form a ratio. Example: If there are 12 boys and 15 girls in a class, the ratio of boys to girls is 12 to 15.

1. ●●●○○○○○
   The ratio of shaded marbles to unshaded marbles is 3 to 5.

2. △△△□□□□
   The ratio of triangles to squares is 3 to 4.

3. 🎈🎈🎈🎈🎈🪁🪁
   The ratio of balloons to kites is 5 to 2.

4. 🪁🪁🎈🎈🎈🎈🎈
   The ratio of kites to balloons is 2 to 5.

5. ●●●○○○○
   The ratio of all the marbles to the unshaded marbles is 7 to 4.

6. ●●●○○○○
   The ratio of shaded marbles to all the marbles is 3 to 7.

7. The ratio 3 to 7 can be written in fractional form as $\frac{3}{7}$. The ratio 3 to 5 can be written as $\frac{3}{5}$.

8. $\frac{6}{7}$ can be written as the ratio 6 to 7.

9. $\frac{4}{9}$ can be written as the ratio 4 to 9.

Write each ratio in fractional form.

10. 2 to 3 = $\frac{2}{3}$

11. 7 to 5 = $\frac{7}{5}$

12. 8 to 11 = $\frac{8}{11}$

13. 14 to 8 = $\frac{14}{8}$

14. XX / XX XX
    This picture shows a ratio of 2 to 4.
    Fractional form: $\frac{2}{4}$

15. XXX / XXX XXX
    Ratio of 3 to 6
    Fractional form: $\frac{3}{6}$

16. X / XX
    Ratio of 1 to 2
    Fractional form: $\frac{1}{2}$

17. XXXXX / XXXXX XXXXX
    Ratio of 5 to 10
    Fractional form: $\frac{5}{10}$

18. $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{5}{10}$ are equivalent fractions. 1 to 2, 2 to 4, 3 to 6, and 5 to 10 are equivalent ratios.

19. There are 15 boys and 30 girls in the chorus. The ratio of boys to girls is 15 to 30. Is the ratio also 1 to 2? yes

20. Name three ratios that are equivalent to $\frac{1}{3}$.
    2 to 6; 3 to 9; 10 to 30.

21. Name four ratios that are equivalent to 3 to 4.
    6 to 8; 9 to 12; 12 to 16; 30 to 40.

2

An equation which states the equality of two ratios is called a proportion.

22. XXXX / XXXX XXXX

$\frac{4}{8} = \frac{1}{2}$

23. XX / XX XX XX

$\frac{2}{6} = \frac{1}{3}$

24. $\frac{3}{4} = \frac{6}{8}$

Is $\frac{3}{4} = \frac{6}{8}$ a proportion? yes

25. $\frac{2}{3} \times \frac{4}{6}$

$2 \times 6 = 3 \times 4$

$12 = 12$

Is $\frac{2}{3} = \frac{4}{6}$ a proportion? yes

26. $\frac{3}{10} = \frac{9}{30}$

$3 \times 30 = 10 \times 9$

$90 = 90$

Is $\frac{3}{10} = \frac{9}{30}$ a proportion? yes

27. $\frac{1}{8} = \frac{2}{5}$

$1 \times 5 = 8 \times 2$

Is $\frac{1}{8} = \frac{2}{5}$ a proportion? no

28. $\frac{3}{5} = \frac{6}{10}$

$30 = 30$

Is $\frac{3}{5} = \frac{6}{10}$ a proportion? yes

Which are proportions? Write yes or no after each problem.

29. $\frac{2}{3} \stackrel{?}{=} \frac{3}{6}$ no

30. $\frac{1}{3} \stackrel{?}{=} \frac{2}{9}$ no

31. $\frac{2}{5} \stackrel{?}{=} \frac{4}{10}$ yes

32. $\frac{4}{5} \stackrel{?}{=} \frac{6}{10}$ no

3

1. ●●●●●○○

What is the ratio of unshaded marbles to shaded marbles? 2 to 5

2. △□□□□

What is the ratio of triangles to squares? 1 to 4

3. Write 2 to 3 in fractional form. $\frac{2}{3}$

4. $\frac{7}{8} =$ 7 to 8

5. Write three ratios equivalent to $\frac{2}{5}$.

4 to 10, 6 to 15, 8 to 20

6. Is $\frac{2}{4} = \frac{1}{2}$ a proportion? yes

If so, why? they are equivalent ratios

Write two ratios to rename the following proportions.

7. $\frac{3}{4} = \frac{15}{20}$ $\frac{6}{8} = \frac{30}{40}$

8. $\frac{5}{6} = \frac{20}{24}$ $\frac{15}{18} = \frac{60}{72}$

9. $\frac{9}{2} = \frac{18}{4}$ $\frac{18}{4} = \frac{36}{8}$

Which are proportions? Write yes or no.

10. $\frac{3}{4} = \frac{5}{8}$ no

11. $\frac{7}{9} = \frac{2}{3}$ no

12. $\frac{5}{8} = \frac{10}{16}$ yes

13. $\frac{3}{5} = \frac{5}{10}$ no

14. $\frac{5}{7} = \frac{10}{14}$ yes

15. $\frac{5}{6} = \frac{8}{12}$ no

4

one row (horizontal, vertical, or diagonal) is the winner.

SUGGESTED RATIOS	SUGGESTED FRACTIONS
24 to 48	$\frac{3}{10}$
6 to 9	$\frac{7}{42}$
5 to 12	$\frac{5}{8}$
3 to 7	$\frac{7}{15}$
2 to 12	$\frac{5}{6}$
3 to 8	$\frac{3}{25}$
15 to 18	$\frac{3}{6}$
20 to 32	$\frac{12}{28}$
21 to 45	$\frac{2}{16}$
6 to 50	$\frac{2}{3}$
2 to 8	$\frac{35}{84}$
30 to 100	
1 to 3	
4 to 32	

2. Have students use their rulers to draw a square with 4-inch sides; then have them draw lines to section the square into sixteen congruent parts (each part will be exactly 1 inch square). When their figures are completed, direct them to use four crayons to color the small squares so that each color represents a proportion of 1 to 4 (the number of colored squares to the total number of squares). The same color may not appear in adjacent squares. An example is shown below, with the letters R, B, Y, and G indicating red, blue, yellow, and green.

Example:

R	B	Y	G
Y	G	R	B
R	B	Y	G
Y	G	R	B

LESSON 3

Meaning of Percent

Approximate time required to complete the lesson: 25 min.

Background Needed

For complete understanding of the concepts presented in this lesson, the student should have some familiarity with fractions and decimals and a knowledge of multiplication of fractions.

In The Lesson

The narrator introduces percents as another way to rename fractions. The student is taught to use the identity element to rename fractions as fractions with denominators of a hundred and then to convert those fractions to decimals or percents. The lesson also includes work on renaming percents and decimals as fractions.

Vocabulary: The following term is introduced and used by the narrator in this lesson.

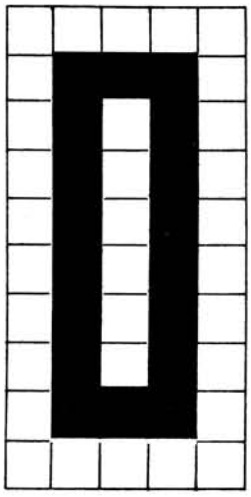
percent: parts per hundred

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate his understanding of the meaning of percent.

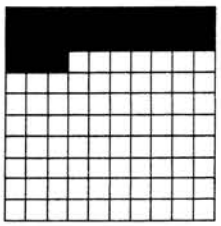
A Step Further

The following activities will help students expand their understanding of topics related to percents.

1. Have students shade 36% of the squares in this rectangle in such a way that the shaded squares are all the same distance from the outside edge.



Formula 1 Math Powerpac E
Lesson 3 Meaning of Percent

1. 

What fraction tells which part of the diagram is shaded?
$\frac{23}{100}$

Percent means parts per hundred.

2. The ratio of 23 to 100 is 23 percent.
The ratio of 47 to 100 is 47 percent.

This is the symbol for percent:
%

3. 17% means 17 percent.
17% means 17 out of 100.
17% written as a fraction is $\frac{17}{100}$.

4. $\frac{31}{100}$ means 31 per hundred.
31 per hundred is 31 percent.
31 percent can be written 31 %.

Write each fraction as a percent using the symbol %.

5. $\frac{12}{100}$ = 12%
6. $\frac{49}{100}$ = 49%
7. $\frac{34}{100}$ = 34%
8. $\frac{7}{100}$ = 7%
9. $\frac{93}{100}$ = 93%
10. $\frac{1}{100}$ = 1%

Write each percent as a fraction.

11. 29% = $\frac{29}{100}$
12. 64% = $\frac{64}{100}$
13. 6% = $\frac{6}{100}$
14. 89% = $\frac{89}{100}$
15. 19% = $\frac{19}{100}$
16. 1% = $\frac{1}{100}$

17. $\frac{23}{100}$ as a decimal is 0.23
18. $\frac{23}{100}$ as a percent is 23%
19. 23% as a decimal is 0.23
20. $\frac{34}{100}$ as a decimal is 0.34
21. $\frac{34}{100}$ as a percent is 34%
22. 34% as a decimal is 0.34

Express each percent as a decimal.

29. 28% = 0.28
30. 7% = 0.07
31. 64% = 0.64
32. 1% = 0.01
33. 98% = 0.98
34. 17% = 0.17

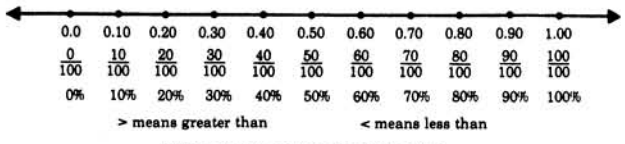
Express each fraction or decimal as a percent.

23. $\frac{89}{100}$ = 89%
24. 0.27 = 27%
25. 0.06 = 6%
26. $\frac{1}{100}$ = 1%
27. 0.92 = 92%
28. $\frac{51}{100}$ = 51%

Express each percent or decimal as a fraction.

35. 0.67 = $\frac{67}{100}$
36. 42% = $\frac{42}{100}$
37. 0.04 = $\frac{4}{100}$
38. 57% = $\frac{57}{100}$
39. 100% = $\frac{100}{100}$
40. 0.59 = $\frac{59}{100}$

2



> means greater than < means less than

Write > or < to make each sentence true.

41. .10 < 17% 43. 70% > 0.69 45. 1% < 0.10

42. 45% < $\frac{50}{100}$ 44. 0.23 > $\frac{20}{100}$ 46. 65% > 0.58

47. $\frac{2}{5}$ = 40%

$\frac{2}{5} \times \frac{20}{20} = \frac{40}{100}$

$\frac{2}{5}$ = 40%

48. $\frac{7}{10} \times \frac{10}{10} = \frac{70}{100}$

$\frac{7}{10}$ = 70 %

49. $\frac{3}{4}$ = ____

$\frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$

$\frac{3}{4}$ = 75 %

50. $\frac{3}{5} = \frac{60}{100}$

$\frac{3}{5}$ = 60 %

51. $\frac{7}{25} = \frac{28}{100}$

$\frac{7}{25}$ = 28 %

52. $\frac{9}{20} = \frac{45}{100}$

$\frac{9}{20}$ = 45 %

53. $\frac{1}{2} = \frac{50}{100}$

$\frac{1}{2}$ = 50 %

54. $\frac{4}{4} = \frac{100}{100}$

$\frac{4}{4}$ = 100 %

Complete these exercises.

1. Percent means per hundred.

2. $\frac{7}{100}$ = 7 %

3. 18% = $\frac{18}{100}$

4. 0.32 = 32 %

5. $\frac{69}{100}$ = 69 %

6. 1% = $\frac{1}{100}$

7. $\frac{9}{100}$ = 9 %

8. $\frac{98}{100}$ = 98 %

In exercises 9, 10, 11, and 12, write < or > to make each sentence true.

9. 42% > 0.37

10. 1% < 0.10

11. 87% > 0.78

12. 5% < 0.50

13. $\frac{2}{5} = \frac{40}{100}$

$\frac{2}{5}$ = 40 %

14. $\frac{3}{4} = \frac{75}{100}$

$\frac{3}{4}$ = 75 %

15. $\frac{11}{20} = \frac{55}{100}$

$\frac{11}{20}$ = 55 %

16. $\frac{13}{25} = \frac{52}{100}$

$\frac{13}{25}$ = 52 %

17. $\frac{9}{10} = \frac{90}{100}$

$\frac{9}{10}$ = 90 %

18. $\frac{1}{2} = \frac{50}{100}$

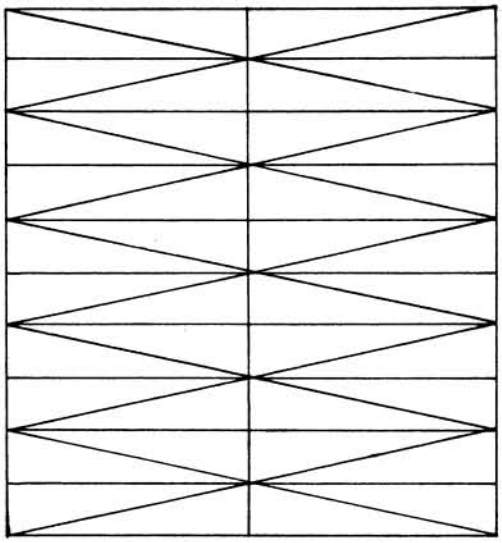
$\frac{1}{2}$ = 50 %

19. 0.34 = 34 %

20. 46% = $\frac{46}{100}$

21. .05 = 5 %

2. Give students a copy of the following figure (explain that all the parts are congruent) and have them solve the problem below.



Assume that a farmer has divided his land into triangular fields as shown in the figure. If he plants corn in just one field, what percent of his land will have corn on it?

1 out of 40 or $2\frac{1}{2}$%

3. To increase understanding and ability to work with percent, create problems which require students to work with very small and very large percents. The following examples offer good practice.

If 1% is 0.01, then $\frac{1}{2}$% is .005.
If 100% is 1, then 300% is 3.
$\frac{1}{2}$% of 48 is what number? .24
200% of 68 is what number? 136
$6\frac{1}{2}$% of 480 is what number? 31.2
520% of 360 is what number? 1872

LESSON 4

Percent of a Number

Approximate time required to complete the lesson: 40 min.

Background Needed

To gain full benefit from this lesson, the student should be familiar with percents and be able to multiply whole, fractional, and decimal numbers.

In The Lesson

Beginning with a review of renaming fractions and decimals as percents, the narrator leads the student through a series of exercises designed to teach him equivalent fractions for commonly used percents. The student is introduced to problem solving with percents and learns that, before performing mathematical operations, percents must be renamed as either fractions or decimals.

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate how well he has understood finding the percent of a number.

A Step Further

The following activities will provide able students an opportunity to extend their knowledge of the use of percents.

1. A card game can provide an interesting exercise in decimal, fraction, and percent equivalents. Use 3- by 5-inch cards to construct eighteen 3-card sets, each of which consists of a fraction, its decimal equivalent, and percent equivalent. Shuffle the 54 cards together and deal out two to each player (from 2 to 6 can play). Place the remaining cards in the center of the table to form an extras pile. The object of the game is to form 'triples'—three equivalent cards; one, a fraction; one, a decimal; and one, a percent. The player to the dealer's left begins the play by drawing a card from the extra pile. If it forms a 'triple,' he may lay it down. If it does not, he must place one of his three cards back in the discard pile. Play may either continue until the first triple is scored or for a specified number of rounds. In the second case, any player who gets a triple must then draw two cards

Formula I Math Powerpac E
Lesson 4 Percent of a Number

Percent means parts per hundred.

1. $\frac{12}{100}$ = 12 %
2. $\frac{93}{100}$ = 93 %
3. $\frac{14}{100}$ = 14 %
4. 0.07 = 7 %
5. 0.23 = 23 %
6. 0.54 = 54 %
7. $\frac{2}{5} = \frac{40}{100}$ $\frac{2}{5}$ = 40 %
8. $\frac{1}{4} = \frac{25}{100}$ $\frac{1}{4}$ = 25 %
9. 25% = $\frac{1}{4}$
10. $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$
11. $\frac{1}{2}$ of 25% is 12½%
12. $12\frac{1}{2}\% = \frac{1}{8}$
13. $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$
14. 25% + $12\frac{1}{2}\%$ = 37½%
15. $37\frac{1}{2}\% = \frac{3}{8}$
16. 50% = $\frac{1}{2}$
17. 50% + $12\frac{1}{2}\%$ = 62½%
18. $\frac{1}{2} + \frac{1}{8} = \frac{5}{8}$
19. $62\frac{1}{2}\% = \frac{5}{8}$
20. 75% = $\frac{3}{4}$
21. 75% + $12\frac{1}{2}\%$ = 87½%
22. $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$
23. $87\frac{1}{2}\% = \frac{7}{8}$
24. $33\frac{1}{3}\% = \frac{1}{3}$
25. $66\frac{2}{3}\%$ = 2/3

Circle the correct fraction for each percent.

26. $12\frac{1}{2}\%$ is: $\frac{1}{4}$, $\frac{3}{8}$, (1/8)
27. $33\frac{1}{3}\%$ is: $\frac{1}{2}$, $\frac{1}{4}$, (1/3)
28. $62\frac{1}{2}\%$ is: (5/8) $\frac{2}{3}$, $\frac{7}{8}$
29. $87\frac{1}{2}\%$ is: $\frac{3}{4}$, (7/8) $\frac{5}{6}$
30. $37\frac{1}{2}\%$ is: $\frac{1}{8}$, $\frac{2}{8}$ (3/8)
31. $66\frac{2}{3}\%$ is: $\frac{3}{8}$, $\frac{1}{3}$ (2/3)

You cannot operate with percents. Before computing, percents must be changed to either fractional or decimal form.

32. $\frac{1}{2}$ of 8 is what number?
$\frac{1}{2} \times 8 =$ 4
33. 50% of 8 is what number?
½ × 8 = 4
34. $\frac{3}{4}$ of 20 is what number?
$\frac{3}{4} \times 20 =$ 15
35. 75% of 20 is what number?
¾ × 20 = 15
36. 60% of 140 is what number?
You may use either $\frac{60}{100}$ or 0.60 for 60%.
.60 × 140 = 84
37. $12\frac{1}{2}\%$ of 88 is what number?
(Think: $12\frac{1}{2}\%$ is what fraction?)
⅛ × 88 = 11
38. 25% of 420 is what number?
¼ × 420 = 105
39. $37\frac{1}{2}\%$ of 160 is what number?
⅜ × 160 = 60
40. 72% of 184 is what number?
.72 × 184 = 132.48
41. 1% of 4,289 is what number?
.01 × 4289 = 42.89

2

42. $62\frac{1}{2}\%$ of 2400 is what number?
$\frac{5}{8} \times 2400 = 1500$

43. 7% of 540 is what number?
$.07 \times 540 = 37.8$

44. $66\frac{2}{3}\%$ of 960 is what number?
$\frac{2}{3} \times 960 = 640$

45. $87\frac{1}{2}\%$ of 840 is what number?
$\frac{7}{8} \times 840 = 735$

46. There are 400 pupils at Greeley School. $37\frac{1}{2}\%$ of the pupils are girls. How many girls attend Greeley School?
$\frac{3}{8} \times 400 = 150$
150 girls attend Greeley School.

47. For the last basketball game, 75% of the 1,640 seats in the gym were filled. How many people attended the game?
$\frac{3}{4} \times 1640 = 1230$
1230 people attended the game.

48. $66\frac{2}{3}\%$ of the 36 football players at Edison High School received awards. How many players received awards?
$\frac{2}{3} \times 36 = 24$
24 football players received awards.

Write as fractions.

1. 25% = $\frac{1}{4}$
2. $12\frac{1}{2}\%$ = $\frac{1}{8}$
3. $33\frac{1}{3}\%$ = $\frac{1}{3}$
4. $87\frac{1}{2}\%$ = $\frac{7}{8}$
5. 75% = $\frac{3}{4}$
6. $37\frac{1}{2}\%$ = $\frac{3}{8}$

Write as decimals.

7. 1% = .01
8. 78% = .78
9. 23% = .23
10. 8% = .08
11. 47% = .47
12. 14% = .14

13. $12\frac{1}{2}\%$ of 480 is what number?
$\frac{1}{8} \times \frac{480^{60}}{1} = 60$

14. 8% of 125 is what number?
$.08 \times 125 = 10$

15. 1% of 2,358 is what number?
$.01 \times 2358 = 23.58$

16. $66\frac{2}{3}\%$ of 993 is what number?
$\frac{2}{3} \times \frac{331}{993} = 662$

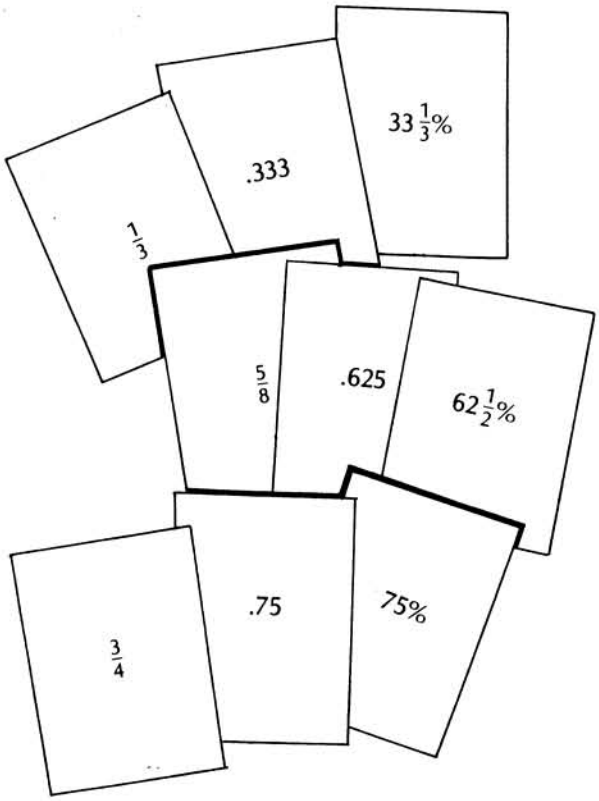
17. $87\frac{1}{2}\%$ of 640 is what number?
$\frac{7}{8} \times \frac{640^{80}}{1} = 560$

18. 75% of 2800 is what number?
$\frac{3}{4} \times \frac{700}{2800} = 2100$

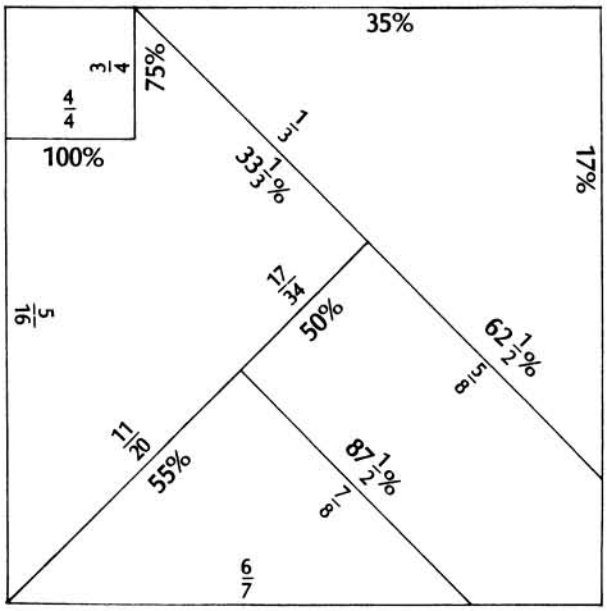
19. $33\frac{1}{3}\%$ of the 360 pupils at Shepard School are in the band. How many band members are there?
$\frac{1}{3} \times \frac{120}{360} = 120$
120 members in the band.

20. 20% of the 160 coins in Bill's collection were gold pieces. How many gold coins did Bill have?
$\frac{1}{5} \times \frac{32}{160} = 32$
Bill had 32 gold coins.

from the extras pile.
Score 10 points for each triple, minus two points for each card left in a player's hand at the end of the game, and minus three points for any triple laid down that is incorrect. An incorrect triple may be pointed out by any player at the time the triple is laid down.



2. Using pieces of lightweight cardboard, construct a puzzle similar to the one shown below for each student. Ask the students to reassemble the pieces to form a square, making sure that each percent is directly opposite its equivalent fraction.



LESSON 5

Using Equations to Find Percent

Approximate time required to complete the lesson: 30 min.

Background Needed

To gain the full benefit from this lesson, the student should have some knowledge of percent, be able to rename fractions and decimals as percents, and write fractions in simplest form.

In The Lesson

To introduce this lesson on problem solving with percents, the narrator reviews the process of changing the English sentence to a math sentence in which *n* represents the answer. The student learns the shortcut to renaming a fraction as a percent; i.e., dividing the numerator by the denominator. A series of exercises is presented to provide practice in finding percents.

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate his understanding of percents.

A Step Further

The following activities will give able students more opportunities to explore percents.

1. Introduce students to percents greater than 100 by explaining that the procedure used in finding such percents is the same as the one they have already learned. Have students find the answers to the following problems in decimal form; then tell them to multiply by 100 to get the equivalent percent.

   Examples:

   120 is what percent of 40?

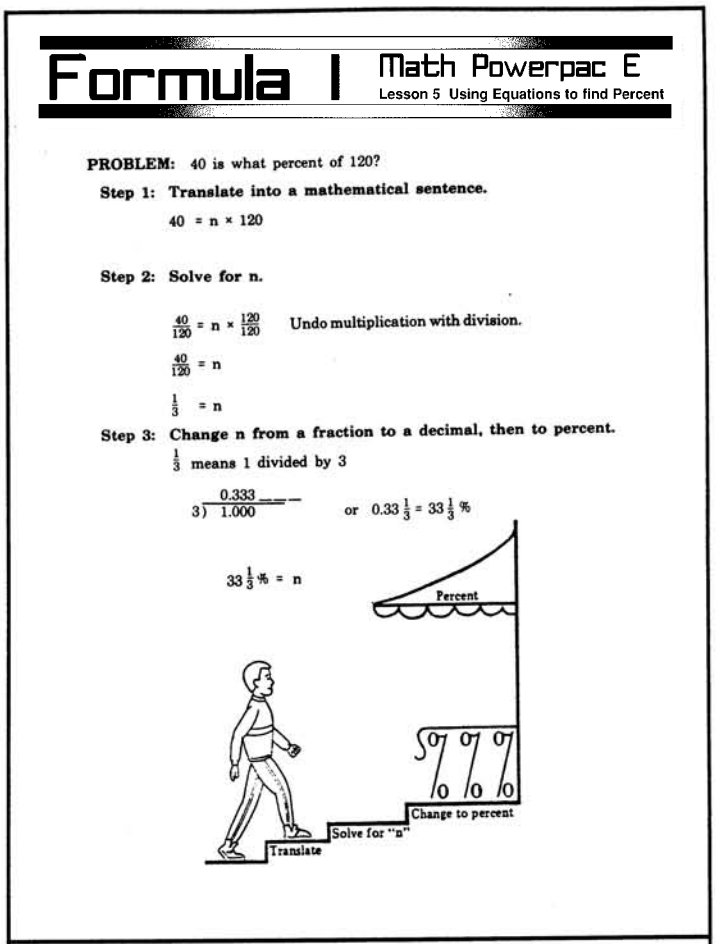
   $$40\overline{)120.00}\quad 3.00$$

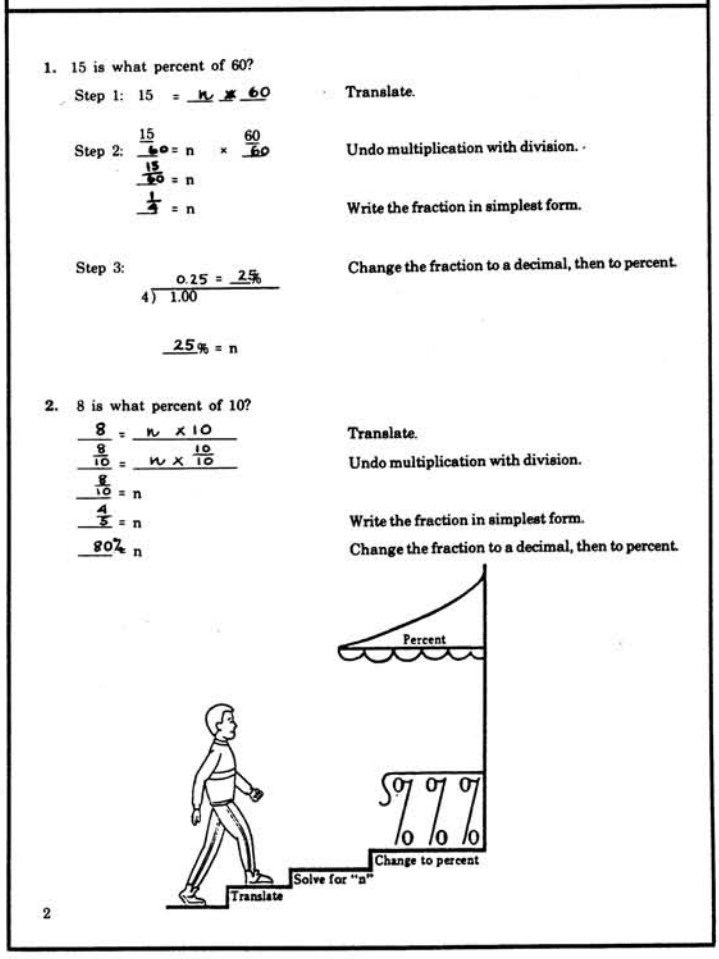
   120 is 300% of 40

   240 is what percent of 60?

   $$60\overline{)240.00}\quad 4.00$$

   240 is 400% of 60





3. 30 is what percent of 240?

$30 = n \times 240$ — Translate.

$\frac{30}{240} = n \times \frac{240}{240}$ — Undo multiplication with division.

$\frac{30}{240} = n$

$\frac{1}{8} = n$ — Write the fraction in simplest form.

$12\frac{1}{2}\% = n$ — Change the fraction to percent.

4. 22 is what percent of 33?

$22 = n \times 33$ — Translate.

$\frac{22}{33} = n \times \frac{33}{33}$ — Undo multiplication with division.

$\frac{22}{33} = n$

$\frac{2}{3} = n$ — Write the fraction in simplest form.

$66\frac{2}{3}\% = n$ — Change the fraction to percent.

5. 35 is what percent of 40?

$35 = n \times 40$ — Translate.

$\frac{35}{40} = n \times \frac{40}{40}$ — Undo multiplication with division.

$\frac{35}{40} = n$

$\frac{7}{8} = n$ — Write the fraction in simplest form.

$87\frac{1}{2}\% = n$ — Change the fraction to percent.

$\frac{1}{8} = 12\frac{1}{2}\%$ $\quad \frac{3}{8} = 37\frac{1}{2}\%$ $\quad \frac{5}{8} = 62\frac{1}{2}\%$ $\quad \frac{7}{8} = 87\frac{1}{2}\%$

$\frac{1}{3} = 33\frac{1}{3}\%$ $\quad \frac{2}{3} = 66\frac{2}{3}\%$

3

1. Solve for n.

$16 = n \times 2$

$8 = n$

2. $\frac{3}{4} = 75\%$

3. "of" means times

4. 9 is what percent of 15?

$9 = n \times 15$ — Translate.

$\frac{9}{15} = n \times \frac{15}{15}$ — Undo multiplication

$\frac{9}{15} = n$ — with division.

$\frac{3}{5} = n$ — Simplify fraction.

$60\% = n$ — Change to percent.

5. 12 is what percent of 32?

$12 = n \times 32$

$\frac{12}{32} = n \times \frac{32}{32}$

$\frac{12}{32} = n$

$\frac{3}{8} = n$

$37\frac{1}{2}\% = n$

6. 120 is what percent of 480?

$120 = n \times 480$

$\frac{120}{480} = n \times \frac{480}{480}$

$\frac{120}{480} = n$

$\frac{1}{4} = n$

$25\% = n$

7. 18 is what percent of 27?

$18 = n \times 27$

$\frac{18}{27} = n \times \frac{27}{27}$

$\frac{18}{27} = n$

$\frac{2}{3} = n$

$66\frac{2}{3}\% = n$

8. 310 is what percent of 620?

$310 = n \times 620$

$\frac{310}{620} = n \times \frac{620}{620}$

$\frac{310}{620} = n$

$\frac{1}{2} = n$

$50\% = n$

9. 48 is what percent of 60?

$48 = n \times 60$

$\frac{48}{60} = n \times \frac{60}{60}$

$\frac{48}{60} = n$

$\frac{4}{5} = n$

$80\% = n$

4

140 is what percent of 84?

$$84\overline{)140.00} = 1.66\frac{2}{3}$$

$$\begin{array}{r} 84 \\ \hline 560 \\ 504 \\ \hline 560 \\ 504 \\ \hline 56 \end{array}$$

140 is $166\frac{2}{3}\%$ of 84

232 is what percent of 28?

$$28\overline{)232.00} = 8.285$$

232 is approximately 829% of 28

2. Tell students that the percentages in the top row of this "magic percent square" add up to 34%. Ask them to see how many other combinations they can find that add up to 34%.

16%	3%	2%	13%
5%	10%	11%	8%
9%	6%	7%	12%
4%	15%	14%	1%

Desired responses from students are as follows: All horizontal rows and vertical columns, and both center diagonal rows add up to 34%. The four corner squares and four center squares also add up to 34%; and so do the opposite pairs of squares (3%, 2% at top center, and 15%, 14% at bottom center; and 5%, 9% at left center, and 8%, 12% at right center). The shortest diagonal rows also add up to 34% (2%, 8%, first diagonal top right, 9%, 15%, first diagonal bottom left; and 5%, 3%, first diagonal top left; and 14%, 12%, first diagonal bottom right).

LESSON 6

More Problems Involving Percent

Approximate time required to complete the lesson: 35 min.

Background Needed

Since the material in this lesson is based on the student's ability to work with sentences involving percent, he should have completed Lessons 2, 3, 4, and 5 before beginning this lesson. He should also have mastered the basic processes of multiplication and division and understand equivalent fractions.

In The Lesson

In this lesson, the narrator guides the student through a series of "everyday situation" problems which involve commissions. He is then taught to solve discount problems using the formula, *rate of discount = amount of discount divided by the original price.*

Special Instructions: The student will need a sheet of scratch paper on which to do computations for some of the problems in the response booklet.

Vocabulary: The following terms are introduced and used by the narrator in this lesson.

commission: a percent of the sale price received by a salesman

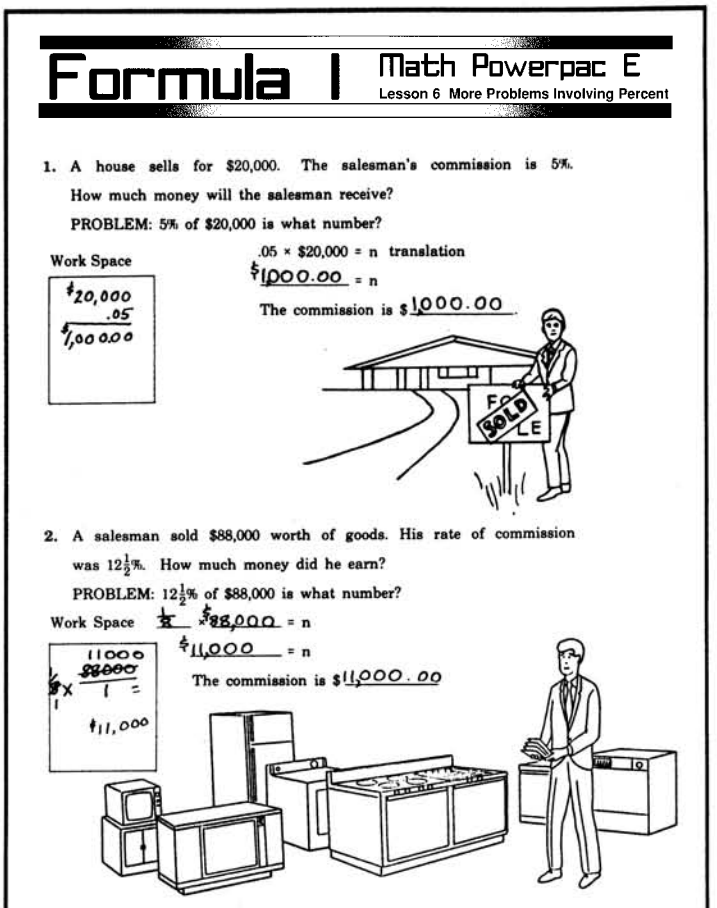
discount: the difference between the regular price and sale price of an item

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate his ability to work with percents.

A Step Further

The following activities will provide students an opportunity to explore the topics presented in this lesson in greater depth.

1. Cut a large advertisement of a discount sale from your local newspaper and have students figure the amounts and rates of discount from the original price of the items listed.
2. Here's a puzzle that will challenge the mental abilities of your able students. Have them color as many blocks as they can without coloring any adjacent blocks. When they have finished, have them compute the percent of the colored parts



Formula 1 Math Powerpac E
Lesson 6 More Problems Involving Percent

1. A house sells for $20,000. The salesman's commission is 5%. How much money will the salesman receive?

PROBLEM: 5% of $20,000 is what number?

Work Space

$20,000 × .05 = $1,000.00

.05 × $20,000 = n translation

$1000.00 = n

The commission is $1,000.00

2. A salesman sold $88,000 worth of goods. His rate of commission was 12½%. How much money did he earn?

PROBLEM: 12½% of $88,000 is what number?

Work Space $\frac{1}{8} \times \$88,000 = n$

$11,000 = n

The commission is $11,000.00

Find the commission for each exercise:

3. Sales: $25,000; rate of commission: 20%

$\frac{1}{5} \times \$25,000 = n$

$5,000 = n

The commission is $5,000.00

Work Space $\frac{1}{5} \times \frac{5000}{\cancel{25000}} = \$5,000$

4. Sales: $56; rate of commission: 37½%

$\frac{3}{8} \times \$56.00 = n$

$21.00 = n

The commission is $21.00

Work Space $\frac{3}{\cancel{8}} \times \frac{7.00}{\cancel{56.00}} = 21.00$

5. Sales: $200; rate of commission: 18%

0.18 × $200 = n

$36.00 = n

The commission is $36.00

Work Space

200
0.18
1600
200
$36.00

6. Sales: $960; rate of commission: 33⅓%

$\frac{1}{3} \times \$960 = n$

$320 = n

The commission is $320.00

Work Space $\frac{1}{3} \times \frac{320}{\cancel{960}} = 320$

7. Sales: $947,000; rate of commission: 1%

.01 × $947,000 = n

$9470.00 = n

The commission is $9470.00

Work Space

$947,000
0.01
$9470.00

2

GOING OUT OF BUSINESS | *Clearance Sale!* | SALE 40% OFF

SAMPLE PROBLEM: $50 bicycles are on sale for $40. What is the *rate of discount*?

Amount saved is discount: $50 - $40 = $10

You save $10.00.

Rate of Discount = amount of discount divided by the original price.

$\frac{10}{50}$ = R (rate of discount)

.20
50)10.00
10.0
000

R = .20 or 20%

When you find *rate* of discount, you are finding a *percent*, so you change from a decimal to a percent.

8. Sale: $6 blouses have been reduced to $4 each.
The discount is $6 - $4 = $2.00
(Rate of discount) R = 2/6 or 1/3
.33 1/3
6)2.00
18
20
18
2
R = 33 1/3 %

9. A coat costs $96, but today it is on sale for $84. What is the rate of discount?
Discount is $96 - $84 = $12.00
R = 12/96 or 1/8
.12 1/2
96)12.00
96
240
192
48
R = 12 1/2 %

10. The regular price of a toy truck is $1.25. But now it is on sale for 50¢. What is the rate of discount?
Discount is $1.25 - .50 = .75
75
3
R = 125/5 or 3/5
.60
125)75.00
750
0
R = 60 %

11. A TV set costs $250. This week it is on sale for $225. Find the rate of discount.
Discount is $250 - $225 = $25
25
1
R = 250/10 or 1/10
.10
250)25.00
250
0
R = 10 %

3

1. Sales: $8,240. Rate of commission is 12½%. What is the commission?
1/8 × $8240 = n
$1,030 = n
Commission is $1030.00

Work Space
1030
8240
1/8 × 1 =
$1030

2. A suit sold for $60. Now it's on sale at $45. Find the rate of discount.
Discount is $60 - $45 = $15
R = 15/60 or .25
R = 25 %

Work Space
.25
60)15.00
120
300
300

3. Sales: $5,000. Rate of commission is 15%. What is the commission?
.15 × $5,000 = n
$750 = n
Commission is $ _____

Work Space
$5000
.15
25000
5000
$750.00

4. A cab driver receives 40% of all fares collected. How much does he receive on $84?
.40 × $84 = n
$33.60 = n

Work Space
$84
.40
$33.60

5. A bicycle that sold for $75 was on sale for $50. What is the rate of discount?
Discount is: $75.00 - $50.00 = $25.00
R = 25/75 or 1/3
R = 33 1/3 %

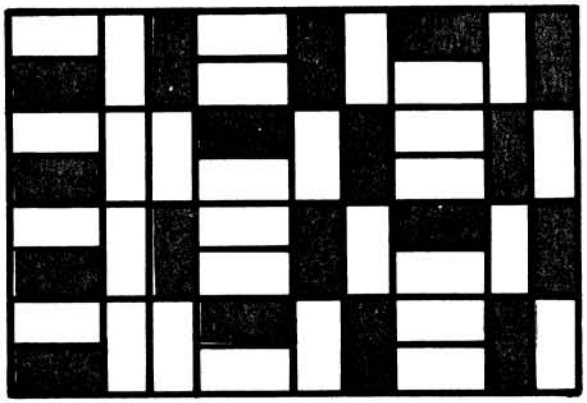
Work Space
.33 1/3
75)25.00
225
250
225
25

6. A roll of film sold for $1.15. The sale price is 92¢. What is rate of discount?
Discount is: $1.15 - .92 = .23
R = 23/115 or 1/5
R = 20 %

Work Space
.20
115)23.00
230
00

4

to the total number of squares. One solution is shown below.



18 out of 48 or $\frac{18}{48} = \frac{3}{8} = 37\frac{1}{2}\%$

3. The following problems offer additional stimulating practice in working with percent.

When Mrs. Blake took attendance, she found that 20% of her pupils were absent. Her list of absentees had five names on it. How many pupils were in Mrs. Blake's class?

There were 25 pupils in Mrs. Blake's class.

Jimmy's older brother was writing a term paper for his college mathematics class. When Jimmy asked him how many pages long the term paper was going to be, this is the answer he got: "I have written 85% of a page and this is $3\frac{1}{2}\%$ of the term paper. There are about 275 words on each page." How many pages did the term paper contain?

There were 24.3 pages in the term paper.

Formula 1 Math Powerpac F

LESSON 1

Writing Math Sentences

Approximate time required to complete the lesson: 40 min.

Background Needed

To benefit fully from this *audio*, the student should be able to perform the basic operations of addition, subtraction, multiplication, and division.

In The Lesson

The narrator explains that the first step in solving story problems is to decide which operation must be used. In the early part of the lesson, the student selects the correct number sentence from two given in the *activity pages*. As the lesson progresses, he is asked to write his own number sentences. Finally, he learns to solve problems which require two steps. Most of the problems in the lesson are read to the student by the narrator.

Special Instructions: The student will need a separate sheet of paper on which to perform computations for some of the problems on the *activity pages*.

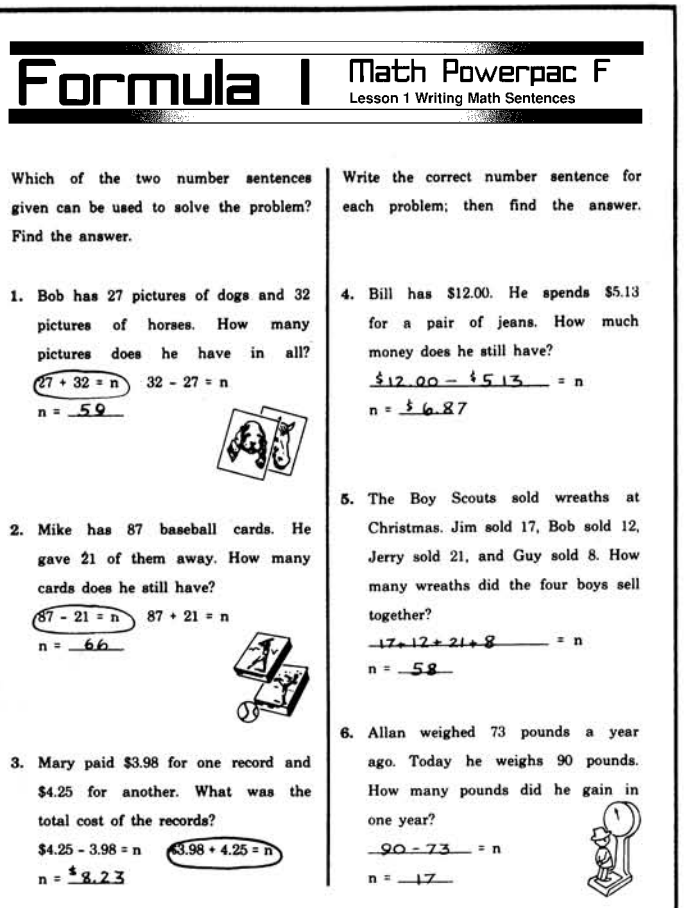
Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate his ability to apply what he has learned from the *audio*.

A Step Further

The following activities will give students an opportunity to expand their understanding of the topics presented in this lesson.

1. Multiple step story problems, in which the unknown number, *n*, appears more than once, present a stimulating challenge for able students. Let them try their skills on the two problems below.

   Two sisters, named Janet and Sue, have a combined age of 34. In 10 years, Sue will be three times as old as Janet is now. How old is each girl?



Formula 1 Math Powerpac F
Lesson 1 Writing Math Sentences

Which of the two number sentences given can be used to solve the problem? Find the answer.

1. Bob has 27 pictures of dogs and 32 pictures of horses. How many pictures does he have in all?
   (27 + 32 = n) 32 - 27 = n
   n = 59

2. Mike has 87 baseball cards. He gave 21 of them away. How many cards does he still have?
   (87 - 21 = n) 87 + 21 = n
   n = 66

3. Mary paid $3.98 for one record and $4.25 for another. What was the total cost of the records?
   $4.25 - 3.98 = n (3.98 + 4.25 = n)
   n = $8.23

Write the correct number sentence for each problem; then find the answer.

4. Bill has $12.00. He spends $5.13 for a pair of jeans. How much money does he still have?
   $12.00 − $5.13 = n
   n = $6.87

5. The Boy Scouts sold wreaths at Christmas. Jim sold 17, Bob sold 12, Jerry sold 21, and Guy sold 8. How many wreaths did the four boys sell together?
   17+12+21+8 = n
   n = 58

6. Allan weighed 73 pounds a year ago. Today he weighs 90 pounds. How many pounds did he gain in one year?
   90 - 73 = n
   n = 17

7. The auditorium at Central Junior High School has 28 rows of seats, with 36 seats in each row. How many seats are there in Central's auditorium?
   Can you use the number sentence, 28 × 36 = n to solve the problem? yes Can you use 36 × 28 = n? yes
   Find the answer. 1008
   36
   28
   288
   72
   1008

8. Canned peaches are stacked 5 cases high in a grocery store. There are 4 rows of cases, with 7 stacks in each row. How many cases of peaches are there?
   Is (5 × 4) × 7 = n a number sentence you can use? yes Write another sentence you could use.
   7x (4x5) = n
   Find the answer. 140

9. The gym at Lincoln Elementary School has 15 rows of bleachers. Each row can seat 48 people. How many people can sit on the bleachers? 15 x 48 = n
   n = 720

10. Thirty-six boys signed up to play little league baseball. There are 9 players on a team. How many teams will there be?
    Is 36 ÷ 9 = n a good sentence? yes
    Could you use n × 9 = 36? yes
    Find the answer. 4

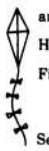
11. How many baseball teams can be formed from 50 players?
    50 ÷ 9 = n
    n = 5 r 5
    5 teams 5 players left over.

12. Bob has 60¢ and pencils cost 8¢ each. How many pencils can he buy?
    60 ÷ 8 = n
    7 pencils 4¢ left over.

13. A storeroom shelf has 8 layers of boxes. Each layer has 10 rows with 7 boxes in each row. How many boxes are there?
    (8 x 10) x 7 = n
    560 boxes

2

14. Sam went to the store with $5.00, and bought eight kites at 45¢ each. How much change did he receive?



First step: Find cost of kites.

8 × 45 = $3.60

Second step: $5.00 − $3.60 = $1.40

15. David delivers 20 papers each morning and 18 papers each evening. How many papers does he deliver in 6 days?

First step: Number of papers delivered each day.

20 + 18 = 38

Second step: 38 × 6 = 228

16. On a recent 3-day trip the Jones family traveled a total of 836 miles. The first day they drove 320 miles, and the second day they drove 285 miles. How far did they drive on the third day?

First step: Distance in 2 days.

320 285 605

Second step: Subtract distance traveled the first 2 days from total miles.

836 605 231

17. Roy wants six 25¢ batteries and a 37¢ wheel for his model airplane. How much will they cost?

First step: 6 × .25 = $1.50

Second step: $1.50 + .37 = $1.87

n = $1.87

18. George has saved 75¢ each week for 5 weeks. How much more does he need to buy a book that costs $4.60?

First step: .75 × 5 = $3.75

Second step: $4.60 − 3.75 = .85

n = 85¢

19. Expenses for a recent birthday party were $1.50 for ice cream, 75¢ for cookies, and $1.50 for soft drinks. The costs were shared equally by three girls. How much did each girl have to pay?

First step:

$1.50 + .75 + $1.50 = $3.75

Second step:

$3.75 ÷ 3 = $1.25

n = $1.25

3

1. Karen sold 42 boxes of Girl Scout cookies and Jane sold 37 boxes. How many boxes did Karen and Jane sell together?

42 + 37 = 79

2. Joe had 317 stamps in his collection, and he gave 90 of them away. How many stamps did he have left?

317 − 90 = 227

3. Judy went shopping with $2.00. She saw three things she wanted: a 98¢ pin, a 79¢ scarf, and a purse that cost $1.50. Which of these items could she buy?

.98 + .79 = $1.77 (pin & scarf)

or the purse

4. Tom had $2.00 when he went to the fair. He spent 55¢ for food and the rest for rides, which cost 35¢ each. How many rides did Tom take?

$2.00 − .55 = $1.45

$1.45 ÷ 35 = n

n = 4 rides

5. June bought six puzzles at 75¢ each. How much change did she receive from $5.00?

.75 × 6 = $4.50

$5.00 − $4.50 = n

n = 50¢

6. Mike cuts 14 lawns a week and he earns $2.25 for each one. How much money does Mike earn in 4 weeks?

$2.25 × 14 = $31.50

$31.50 × 4 = $126.00

7. If a jet flies at 535 miles per hour, how far will it travel in 3 hours?

535 × 3 = 1605 mi.

8. Jim weighed 120 pounds 3 months ago, and today he weighs 132 pounds. How much weight has Jim gained?

132 − 120 = 12 lbs.

4

let n = Janet's age

then $34 - n$ = Sue's age

$(34 - n) + 10$ = Sue's age ten years from now

$(34 - n) + 10 = 3n$

$34 - n + 10 = 3n$

$44 - n = 3n$

$(44 - n) + n = 3n + n$

$44 = 4n$

$11 = n$

Janet is 11 years old.

Sue is 34 − 11 or 23 years old.

On the first day of school, there were three times as many girls as there were boys in the 10 o'clock recess. On the second day, when there were three less boys and three more girls, there were 5 times as many girls as boys. How many boys and how many girls were there in the 10 o'clock recess on the first day? Hint: Let n = the number of boys on the first day.

n = boys on first day

so $3n$ = girls on first day

$n - 3$ = boys on second day

$3n + 3$ = girls on second day

$5(n - 3) = 3n + 3$

$5n - 15 = 3n + 3$

$5n = 3n + 18$

$5n - 3n = 18$

$2n = 18$

$n = 9$

There were 9 boys in 10 o'clock recess on the first day and 3 times 9 or 27 girls.

2. **Give students mathematical sentences and have them create stories that may be solved by the sentences.**

$2 \times (n - 7) = 16$

$n + 7 = 18$

$3n + 5 = 440$

$(96 + 6) \times n = 64$

$(150 \times 2) - 4 = n$

LESSON 2

Number Systems with Bases other than 10

Approximate time required to complete the lesson: 25 min.

Background Needed

Before beginning this lesson, the student should have an understanding of the decimal system and know the meaning of place value.

In The Lesson

After a brief review of the decimal system, the narrator uses sets of stars to illustrate a number system with a base of five. A place value chart for base five is then introduced to show the ones, fives, and twenty-fives places. The narrator gives the student practice in reading one- two- and three-place numerals in base five and renaming them in base ten. The lesson also includes problems involving bases four, six, seven, and eight.

Vocabulary: The following term is introduced and used by the narrator on this ***audio.***

base: the value (to the left of the ones place) upon which a numeration system is based

Evaluation: **The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate how well he has understood bases other than 10.**

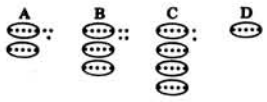
A Step Further

The following activities will provide able students an opportunity to explore bases other than ten in greater depth.

1. Challenge your students with "brainbuster" multiplication problems in bases other than ten. Some examples follow:

$$\begin{array}{r} 435_{six} \\ \times\ 104_{six} \\ \hline 3032 \\ 1000 \\ 435 \\ \hline 50532_{six} \end{array}$$

Formula 1 Math Powerpac F
Lesson 2 Number Systems with Bases Other Than 10



1. PLACE VALUE CHART

	Fives	Ones
A	2	3
B	3	4
C	4	2
D	1	0

2. 23_{five} is read "two three base five."
2 sets of 5 and 3 ones
$(2 \times 5) + (3 \times 1)$
$10_{ten} + 3_{ten}$
$23_{five} = 13_{ten}$

3. 34_{five} is read "three four base five."
3 sets of 5 and 4 ones
$(3 \times 5) + (4 \times 1)$
$15_{ten} + 4_{ten}$
$34_{five} = 19_{ten}$

4. 42_{five} = 4 sets of 5 and 2 ones.
$(4 \times 5) + (2 \times 1)$
$20_{ten} + 2_{ten}$
$42_{five} = 22_{ten}$

5. 10_{five} = 1 set of 5 and 0 ones.
$(1 \times 5) + (0 \times 1)$
$5_{ten} + 0_{ten}$
$10_{five} = 5_{ten}$

6. PLACE VALUE CHART

	Twenty-fives	Fives	Ones
	5 × 5	5	1
A	1	0	0
B	1	2	4

In base five our numerals are 0, 1, 2, 3, and 4. We group in sets of five. Five sets of five form a new unit called *twenty-five.* 100_{five} is read "one zero zero base five."

7. $124_{five} = (1 \times 25) + (2 \times 5) + (4 \times 1)$
$= 25_{ten} + 10_{ten} + 4_{ten}$
$124_{five} = 39_{ten}$

8. $231_{five} = (2 \times 25) + (3 \times 5) + (1 \times 1)$
$= 50_{ten} + 15_{ten} + 1_{ten}$
$= 66_{ten}$

2

9. 24_{six} is read "two four base six."

$24_{six} = (2 \times \underline{6}) + (\underline{4} \times 1)$

$= \underline{12}_{ten} + \underline{4}_{ten}$

$= \underline{16}_{ten}$

10. 53_{eight} is read "five three base eight."

$\underline{5}$ sets of eight and $\underline{3}$ sets of one

$53_{eight} = (5 \times \underline{8}) + (3 \times \underline{1})$

$= \underline{40}_{ten} + \underline{3}_{ten}$

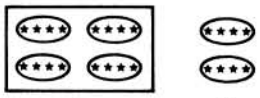
$= \underline{43}_{ten}$

11. $43_{seven} = (\underline{4} \times 7) + (3 \times \underline{1})$

$= \underline{28}_{ten} + \underline{3}_{ten}$

$= \underline{31}_{ten}$

12.



1 2 3

$\underline{1}$ set of sixteen, $\underline{2}$ sets of $\underline{4}$, $\underline{3}$ sets of $\underline{1}$

$123_{four} = (1 \times \underline{16}) + (2 \times \underline{4}) + (3 \times \underline{1})$

$= \underline{16}_{ten} + \underline{8}_{ten} + \underline{3}_{ten}$

$= \underline{27}_{ten}$

1. In 34_{five} the word "five" tells you the numeral is written in base $\underline{five}$.

2. $34_{five} = (3 \times \underline{5}) + (\underline{4} \times 1)$

$= \underline{15}_{ten} + \underline{4}_{ten}$

$= \underline{19}_{ten}$

3. 10_{five} $\underline{5}_{ten}$

4. 5 sets of five is $\underline{25}$.

5. $132_{five} = (\underline{1} \times 25) + (3 \times \underline{5}) + (\underline{2} \times \underline{1})$

$= \underline{25}_{ten} + \underline{15}_{ten} + \underline{2}_{ten}$

$= \underline{42}_{ten}$

6. $52_{six} = (5 \times \underline{6}) + (2 \times \underline{1})$

$= \underline{30}_{ten} \quad \underline{2}_{ten}$

$= \underline{32}_{ten}$

7. 

What base? $\underline{3}$

Write the numeral. $\underline{22}$

8. 4 sets of four is $\underline{16}$

$213_{four} = (2 \times \underline{16}) + (1 \times \underline{4}) + (3 \times \underline{1})$

$= \underline{32}_{ten} + \underline{4}_{ten} + \underline{3}_{ten}$

$\underline{39}_{ten}$

$281_{nine} = 3223_{four}$

$\times 132_{four} = 132_{four}$

13112

23001

3223

1232022 four

or

281_{nine}

$\times 33_{nine}$

863

863

10603 nine

2. To prepare students for operations involving numbers with bases other than 10, have them fill in addition and multiplication tables for *base seven*. The completed tables are shown below.

BASE SEVEN

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	10
2	2	3	4	5	6	10	11
3	3	4	5	6	10	11	12
4	4	5	6	10	11	12	13
5	5	6	10	11	12	13	14
6	6	10	11	12	13	14	15

×	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	4	6	11	13	15
3	0	3	6	12	15	21	24
4	0	4	11	15	22	26	33
5	0	5	13	21	26	34	42
6	0	6	15	24	33	42	51

3. Present the following problems in base seven and let students refer to the charts above as an aid in finding solutions.

Examples:

a. $6_{seven} + 3_{seven} = 12_{seven} = 9_{ten}$

b. $4_{seven} + 6_{seven} = 13_{seven} = 10_{ten}$

c. $6_{seven} \times 4_{seven} = 33_{seven} = 24_{ten}$

LESSON 3

Nonmetric Geometry

Approximate time required to complete the lesson: 35 min.

In The Lesson

The narrator introduces and explains the basic geometric concepts of point, line, line segment, ray, angle, and plane. As the lesson progresses, the circle, radius, chord, and diameter are also illustrated and explained by the narrator, and then put into practice by the student.

Special Instructions: Students will need a ruler and two well- sharpened pencils for the exercises on the *activity pages.*

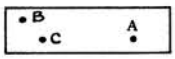
Vocabulary: The following terms and symbols are introduced and used by the narrator in this lesson.

- **point:** a location in space
- **line:** a set of points extending in two directions without end; indicated by $\leftrightarrow$
- **line segment:** all the points on a line between and including the two endpoints; indicated by ——
- **endpoints:** two points between which all the other points of a line segment lie
- **ray:** a set of points which has one endpoint and extends indefinitely in one direction; indicated by $\rightarrow$
- **angle:** formed by two rays with a common endpoint; indicated by $\angle$
- **vertex:** the common endpoint at the intersection of the two rays
- **plane:** a flat surface which extends indefinitely in all directions
- **circle:** a shape made from a set of points, each of which is the same distance from a given point called the center
- **radius:** a line segment whose endpoints are the center of a circle and a point on the circle
- **chord:** a line segment with both endpoints on the circle
- **diameter:** a line segment that passes through the center of a circle and has both its endpoints on the circle

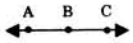
Evaluation: The student's performance on page 4 of the *activity pages,* which he completes after the *audio* has finished playing, will indicate his understanding of nonmetric geometry, as presented by the *audio.*

Formula 1 Math Powerpac F
Lesson 3 Nonmetric Geometry

POINT — **A point is a place in space.**

1. 

LINE — **A line is named by two labeled points.**



2. $\overleftrightarrow{AB}$ means line AB.
$\overleftrightarrow{BC}$ means line BC

LINE SEGMENT — **A line segment is all the points on a line between and including the two endpoints.**

3. $\overline{DE}$ means line segment DE.
$\overline{ED}$ means line segment ED

4. Is line segment $\overline{DE}$ the same as line segment $\overline{ED}$? yes

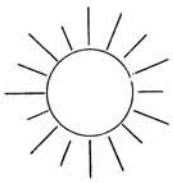
5. Is $\overleftrightarrow{AC}$ the same as $\overleftrightarrow{CA}$? yes

6. Use your pencil and ruler to draw three lines through point F.

7. Draw a line through points G and H.

8. How many lines can pass through one given point? any number

9. How many lines can pass through two given points? one



ray AB

A ray is part of a line. It has only one endpoint.

10. $\overrightarrow{AB} = \overrightarrow{BA}$? no

11. 

$= \overrightarrow{PS}$

12. 

$= \overrightarrow{CD}$

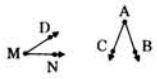
13. 

$= \overrightarrow{EF}$

14. 

$= \overrightarrow{HG}$

Angles are formed by two rays with one common endpoint.



15. $\angle$ DMN means angle DMN. What is the common endpoint in $\angle$ DMN?
M

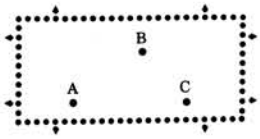
The common endpoint is called the vertex.

16. What is the vertex of $\angle$ CAB?
A

17. Is $\angle$ CAB the same as $\angle$ BAC?
yes

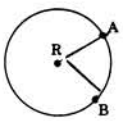
18. $\angle$ DMN may also be called NMD

19. Point M in $\angle$ DMN is called the vertex.



A plane is a flat surface which extends indefinitely.

2



A circle is a shape made from a set of points, each of which is the same distance from a given point called the center.

20. What is the name of the circle above? R

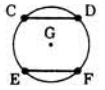
21. With a ruler, draw $\overline{RA}$.

22. Endpoint R is the center of the circle and endpoint A is on the circle.

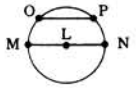
A radius is a line segment with one endpoint at the center of the circle and the other endpoint on the circle.

23. Use a ruler to draw $\overline{RB}$ in circle R.

24. $\overline{RB}$ is a radius of circle R.

25. 

Is $\overline{CD}$ a radius of circle G? no

26. 

Is $\overline{OP}$ a radius of circle L? no

A chord has both endpoints on the circle.

27. Is $\overline{EF}$ a chord? yes

28. Is $\overline{MN}$ a chord? yes

29. $\overline{MN}$ goes through what three given points? M, L, N

A diameter is a chord that has the center of the circle as one of its points.

30. Is $\overline{MN}$ a diameter? yes

31. How many radii (plural for radius) are in a diameter? 2

3

What do these symbols mean?

1. $\overleftrightarrow{AB}$ = line AB

2. $\overrightarrow{AB}$ = ray AB

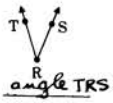
3. $\overline{AB}$ = line segment AB

4. A dot represents a point in geometry.

5. Name the endpoints of $\overline{EF}$.
E and F

6. How many lines can be drawn through two given points? one

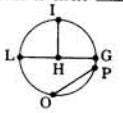
7. How many points are needed to name a line? two

8. 

$\angle$ angle TRS
Vertex R

9. Draw ray ST.


10. Draw angle ABC.


11. This is circle H.


12. $\overline{IH}$ is called a radius.

13. $\overline{OP}$ is called a chord.

14. $\overline{LG}$ is called a diameter.

15. $\overline{LH}$ is called a radius.

16. $\overline{HG}$ is called a radius.

Brain Twister:

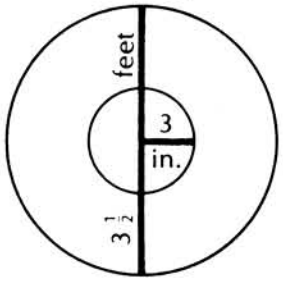

Draw four chords that divide this circle into 10 interiors.

4

The following activities are suggested to provide able students an opportunity to expand their understanding of nonmetric geometry.

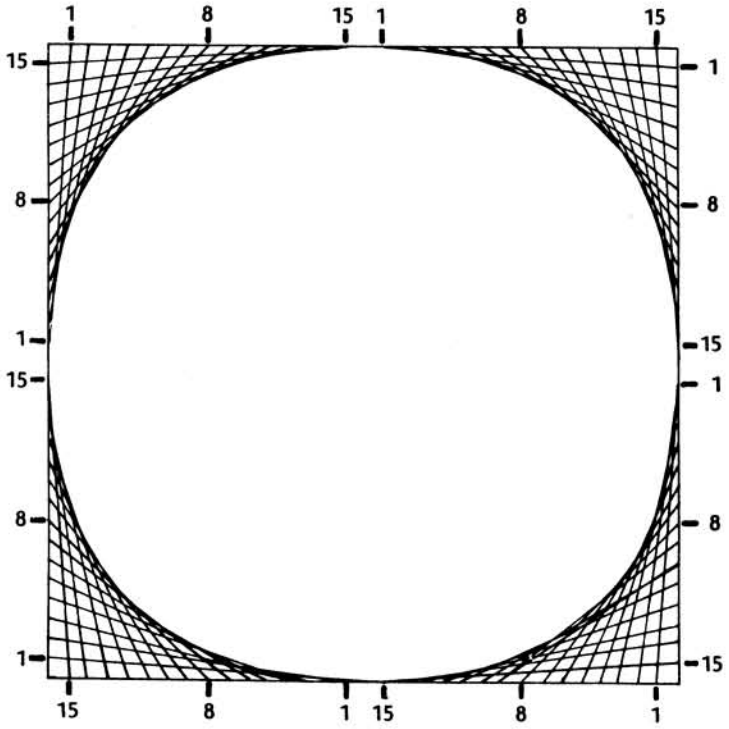
1. Present students with the following problem which deals with radii and diameters. Encourage them to draw a diagram to find the solution.

   Jack had all the parts for a wagon wheel except the spokes, which he decided to make. If the hub had a radius of 3 inches and the wheel rim had a diameter of $3\frac{1}{2}$ feet, how long must Jack make the spokes?



$$\text{wheel radius} = 3\tfrac{1}{2} \div 2 = 1\tfrac{3}{4}\text{ ft.}$$
$$\text{hub radius} = 3\text{ inches} = \tfrac{1}{4}\text{ ft.}$$
$$\text{spoke} = 1\tfrac{1}{2}\text{ ft.}$$

2. Tell students that they can approximate a circle with straight lines. Have each student draw a 4-inch square and mark off each side at ⅛-inch intervals. Starting with the point next to the midpoint, line segments should be drawn, in each quarter, from point to corresponding point in the manner shown below.



LESSON 4

Perimeter, Area, and Metric System

Approximate time required to complete the lesson: 30 min.

Background Needed

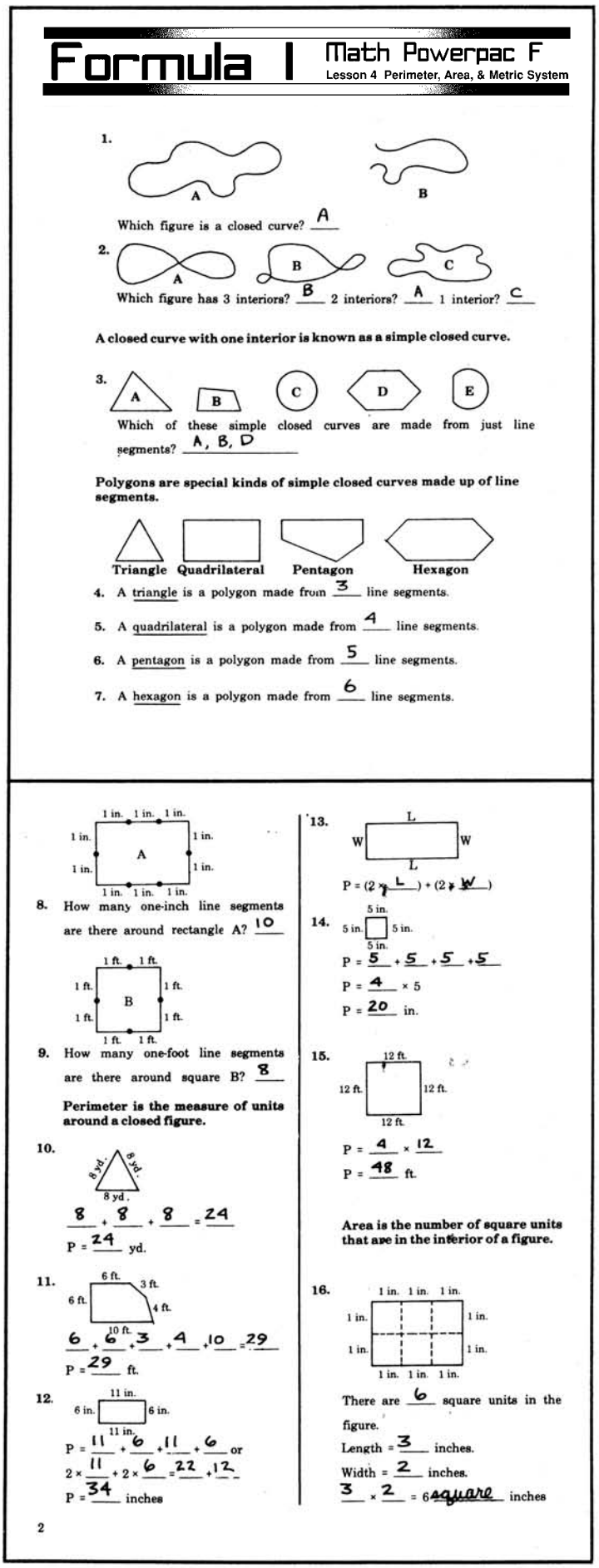
This lesson involves computing perimeter and area. The student should therefore be proficient in addition and multiplication before working with the *audio.*

In The Lesson

Following an explanation of simple closed curves and polygons, the narrator teaches the student to find perimeter by adding the units around a figure. This is followed by a discussion of shortcuts for finding the perimeter of rectangles and squares. Area is introduced next, as the narrator leads the student to discover yet another shortcut for determining the area of rectangles and squares. The metric system is also presented in the lesson, and the student is given practice in finding perimeter and area using the meter as a basis for measurement.

Vocabulary: The following terms and formulas are introduced and used by the narrator on the *audio.*

- **simple closed curve:** a set of points on a path that can be traced continuously from starting point back to starting point without intersecting itself
- **interior:** the set of points inside a simple closed curve
- **polygon:** a simple closed curve that is a union of line segments
- **quadrilateral:** a polygon that is the union of four line segments
- **pentagon:** a polygon that is the union of five line segments
- **hexagon:** a polygon that is the union of six line segments
- **perimeter:** the distance around a simple closed curve
- **area:** the number of units in the interior of a simple closed curve
- **meter:** the basic unit of length in the metric system, about 39 inches long
- **centimeter:** one hundredth of a meter, about .4 inches



Formula 1 Math Powerpac F
Lesson 4 Perimeter, Area, & Metric System

1.

Which figure is a closed curve? A

2.

Which figure has 3 interiors? B 2 interiors? A 1 interior? C

A closed curve with one interior is known as a simple closed curve.

3.

Which of these simple closed curves are made from just line segments? A, B, D

Polygons are special kinds of simple closed curves made up of line segments.

4. A triangle is a polygon made from 3 line segments.
5. A quadrilateral is a polygon made from 4 line segments.
6. A pentagon is a polygon made from 5 line segments.
7. A hexagon is a polygon made from 6 line segments.

8. How many one-inch line segments are there around rectangle A? 10

9. How many one-foot line segments are there around square B? 8

Perimeter is the measure of units around a closed figure.

10.

8 + 8 + 8 = 24
P = 24 yd.

11.

6 + 6 + 3 + 4 + 10 = 29
P = 29 ft.

12.

P = 11 + 6 + 11 + 6 or
2 × 11 + 2 × 6 = 22 + 12
P = 34 inches

13.

P = (2 × L) + (2 × W)

14.

P = 5 + 5 + 5 + 5
P = 4 × 5
P = 20 in.

15.

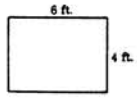
P = 4 × 12
P = 48 ft.

Area is the number of square units that are in the interior of a figure.

16.

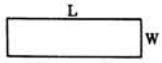
There are 6 square units in the figure.
Length = 3 inches.
Width = 2 inches.
3 × 2 = 6 square inches

2

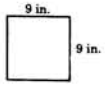
17. 

A = 6 × 4

A = 24 sq. ft.

18. 

A = L × W

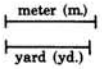
19. 

A = 9 × 9

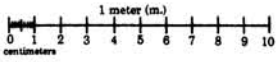
A = 81 sq. in.

20. 

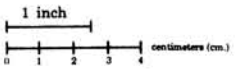
A = S × S

21. 

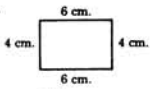
Which is longer, the meter or the yard? meter

22. 

1 meter = 100 centimeters

23. 

1 inch is more than 2 cm. and less than 3 cm.

24. 

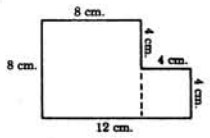
P = 20 cm.

A = 6 × 4 or 24 sq. cm.

25. 

P = 48 m.

A = 12 × 12 or 144 sq. m.

26. 

P = 40 cm.

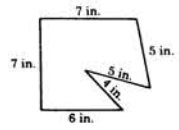
A = 64 + 16 or 80 sq. cm.

3

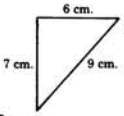
1. The measure of units around a closed figure is called the perimeter.

2. The measure of square units that make up the surface of a figure is called the area.

Study each figure carefully, then fill in the blanks.

3. 

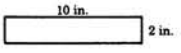
P = 34 in.

4. 

P = 22 cm.

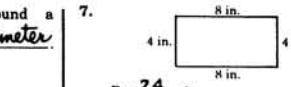
5. 

P = 14 yd.

6. 

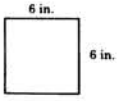
P = 24 in.

A = 20 sq. in.

7. 

P = 24 in.

A = 32 sq. in.

8. 

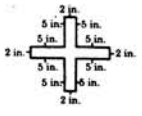
P = 24 in.

A = 36 sq. in.

9. Can the perimeter of two figures be the same and the area of the same two figures be different? yes

10. Which is longer, a centimeter or an inch? inch

11. Which is longer, a meter or a yard? meter

12. 

Area of the figure is

24 + 10 + 10 = 44 sq. in.

Perimeter is 40 + 8 = 48 in.

4

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should enable you to determine how well he understands perimeter, area, and the metric system.

A Step Further

The following activities will provide able students an opportunity to expand their understanding of the concepts covered in this lesson.

1. Have students draw and label as many rectangles as they can devise with a perimeter of 32 inches (the dimensions should be in whole numbers only). When they have finished their drawings, ask them to find the area of each rectangle.

2. Present the students with this perimeter and area puzzle.

   The city was given a piece of land four blocks long and four blocks wide for a sports park. The park commission decided to use one-fourth of the land for a municipal stadium. The remaining land was to be divided according to this plan: four pieces of land, each one-half the size of the stadium, for softball, football, roller skating, and a parking lot; four pieces of land, each half the size of that allotted to softball, for archery, tennis, miniature golf, and swimming. Draw a map showing how the land was divided and label each portion.

   The solution is shown below.

<table>
<tr><td>archery
1 square block</td><td>swimming
1 square block</td><td rowspan="2">softball
2 square blocks</td><td rowspan="2">football
2 square blocks</td></tr>
<tr><td>tennis
1 square block</td><td>min. golf
1 square block</td></tr>
<tr><td>parking lot
2 square blocks</td><td>roller skating
2 square blocks</td><td colspan="2">stadium
4 square blocks</td></tr>
</table>

LESSON 5

Area and Volume

Approximate time required to complete the lesson: 18 min.

Background Needed

To successfully participate in this lesson, the student must be able to multiply with two-digit numbers.

In The Lesson

The student is introduced to the rectangular prism and to volume. After listing the standard cubic units (cubic inch, cubic foot, and cubic yard), the narrator teaches the rule for finding the volume of any rectangular prism—length times width times height. The student also works with cubic centimeters and is introduced to the cube as a special rectangular prism which has equal length, width, and height.

Vocabulary: The following terms are introduced and used by the narrator in this lesson:

rectangular prism: a three-dimensional figure whose base and sides are rectangular

cube: a special rectangular prism in which width, length, and height are equal

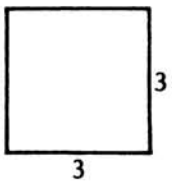
volume: the measure of the interior of any three-dimensional figure

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate his ability to compute volume.

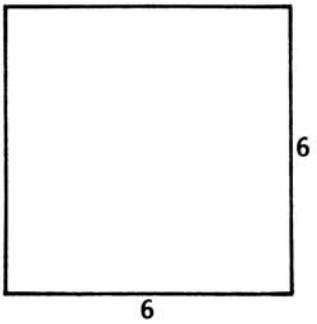
A Step Further

The following activities will give able students an opportunity to expand their understanding of the concepts presented on the *audio*.

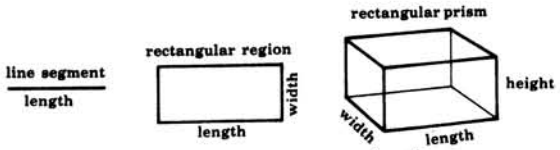
1. Give students the following questions and diagrams and have them compute the answers.



What is the ratio of the area of the smaller square to the area of the larger?

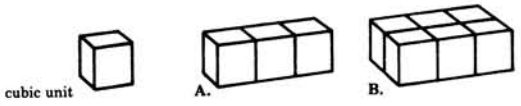


9 to 36 or 1 to 4



1. A line segment has only one dimension, called length.
   A rectangular region has two dimensions, called length and width.
   A rectangular prism has three dimensions, called length, width and height.

2. **The space enclosed by a closed surface, such as a box, is called a solid region. The measure of a solid region is called the volume of the region.**

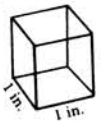


What is the volume, in cubic units, of Figure A? 3
of Figure B? 6

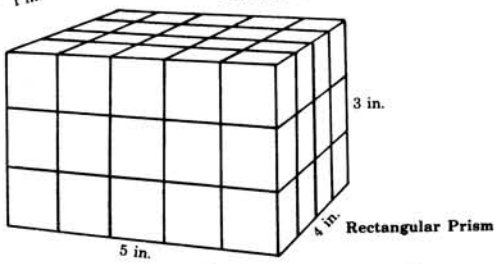
3. A. _____ line segment unit B. square unit C. cubic unit

Which of the units shown above is used to measure length? A
area? B volume? C

4. The standard cubic units are cubic inch, cubic foot, cubic yard, etc.



A cubic inch has a length, width, and height of 1 inch.
A cubic foot has a length, width, and height of 1 foot.



5. The length of the prism is 5 inches, the width is 4 inches, and the height is 3 inches.

The volume of a rectangular prism is found by multiplying the length times the width times the height.

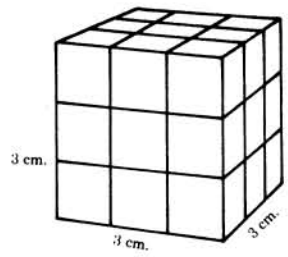
6. To fill the bottom layer of the prism, how many cubic inches would you need? 5 × 4 × 1 = 20 cubic inches.

7. How many layers are there altogether in the prism? 3

8. How many cubic inches would be needed to fill the entire prism? 5 × 4 × 3 = 60 cubic inches.

9. Volume is always expressed in cubic units.

2



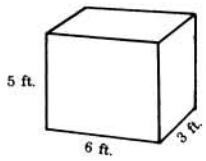
Cube

10. The length of the cube is 3 centimeters, the width is 3 centimeters, and the height is 3 centimeters.

11. How many cubic centimeters will it take to fill the bottom layer of the cube? 3 × 3 × 1 = 9 cubic centimeters.

12. How many layers are there altogether? 3

13. The volume of the cube is 3 × 3 × 3 = 27 cubic centimeters.



Prism

14. What is the area of the base of the prism?
6 × 3 = 18 square feet.

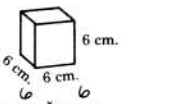
15. The volume of the prism is: 6 × 3 × 5 = 90 cubic feet.

3

1. A rectangular prism has three dimensions: (a) length (b) width (c) height.
2. The measure of a solid region is called its area.
3. A geometric figure that has equal length, width, and height is called a cube.
4. Volume is always expressed in cubic units.

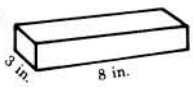
Find the volume. Don't forget to give the unit of measurement with your answers.

5.



6 × 6 × 6
Volume = 216 cubic cm.

6.



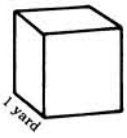
8 × 3 × 2
Volume = 48 cubic in.

7. A rectangular prism has a length of $5\frac{1}{2}$ feet, a width of 6 feet, and a height of 3 feet. What is its volume?
$5\frac{1}{2}$ × 6 × 3
Volume = 99 cubic ft.

8. Find the volume of a prism that has a length of 2.25 centimeters, a width of 3.5 centimeters, and a height of 8 centimeters.
$2\frac{1}{4}$ × $3\frac{1}{2}$ × 8
Volume = 63 cubic cm.

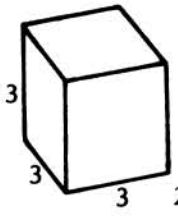
9. 1 yard = 3 feet.

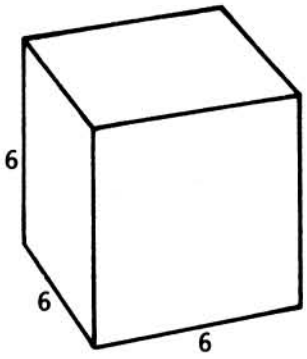
10. How many cubic feet make a cubic yard?
3 × 3 × 3 = 27 cu. ft.



4

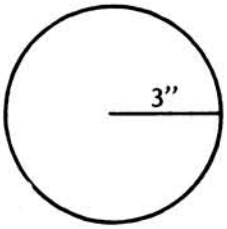
What is the ratio of the smaller cube's volume to the larger?





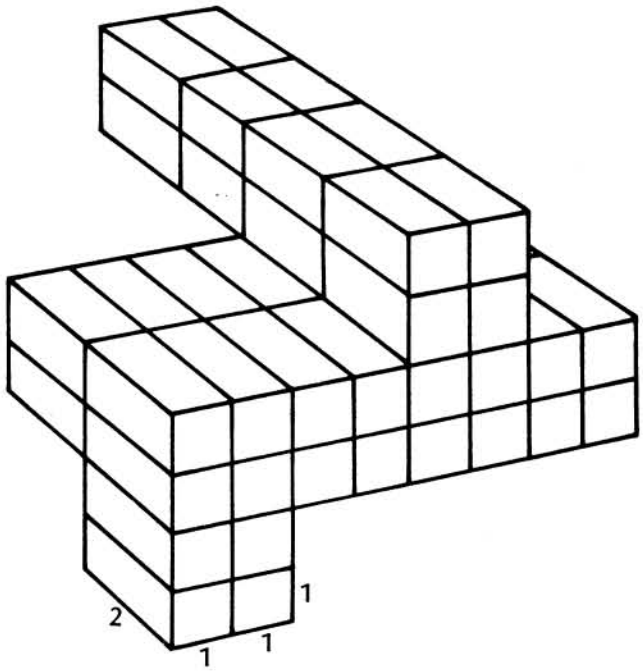
27 to 216 or 1 to 8

What is the ratio of the areas of two circles where the radius of one circle is three times as large as the smaller circle?



28.2744 to 3.1416 or 9 to 1

2. **Ask students to compute the volume of this "miscellanogon." Before they begin, tell them that the figure is composed of 1-inch by 1-inch by 2-inch blocks.**



Volume: 104 cubic inches

LESSON 6

Averages, Graphs, and Tables

Approximate time required to complete the lesson: 22 min.

Background Needed

Before beginning this lesson, the student should possess a basic understanding of the four fundamental mathematical operations.

In The Lesson

The lesson begins with an introduction of the two-step method by which averages are computed. ***audio*** directed problems in the response booklet allow the student to practice this procedure. Tables and circle and bar graphs are then presented as an efficient means of reading statistical information. Throughout the lesson, the student works with problems which are based on information contained in graphs and tables in the booklet.

Special Instructions: The student will need a sheet of scratch paper for computing some of the problems in the response booklet.

Vocabulary: The following terms are introduced and used by the narrator in this lesson.

- **average:** the statistical mean of a sequence of numbers; computed by adding the numbers and then dividing by the number of items in the series
- **bar graph:** a graphic illustration in which statistics can be compared by relative lengths of bars
- **circle graph:** a graphic illustration of data shown by means of a segmented circle
- **table:** a systematic arrangement of statistical data

Evaluation: The student's performance on page 4 of the *activity pages*, which he completes after the *audio* has finished playing, should indicate how well he has understood the *audio* instruction on computing averages and using graphs and tables.

A Step Further

The following activities will allow able students to extend their understanding of the topics presented by the ***audio***.

Formula I Math Powerpac F
Lesson 6 Averages, Graphs, and Tables

1. John's father's golf scores for 6 weeks were: 75, 82, 79, 81, 83, and 86. He wanted to find his average score.
   First step: 75 + 82 + 79 + 81 + 83 + 86 = 486
   Second step: 486 ÷ 6 = 81

2. How many of John's father's scores were above his average? 3

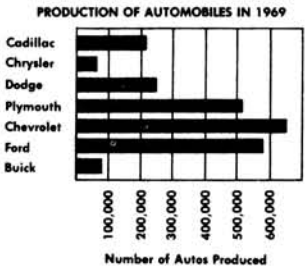
3. Did John's father ever make an average score? yes

4. Mary earned the following grades on mathematics tests: 92, 88, 84, 98, 93.
   What was Mary's average score?
   First step 92 + 88 + 84 + 98 + 93 = 455
   Second step 455 ÷ 5 = 91

5. Rainfall in June was 12.4 inches; in July, 3.82 inches; and, in August, 7.24 inches.
   What was the average rainfall for the 3 months?
   First step 12.4 + 3.82 + 7.24 = 23.46
   Second step 23.46 ÷ 3 = 7.82

BAR GRAPH

The bar graph allows us to quickly compare statistical information.



6. About how many Fords were made in 1969? 575,000

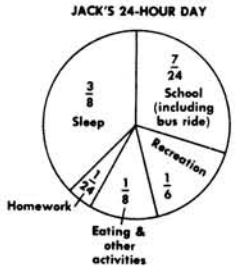
7. Were there more Dodges or Fords produced in 1969? Fords

8. About how many Cadillacs were produced in 1969? 200,000

9. Which make was lowest in production for 1969? Chrysler

CIRCLE GRAPH

A circle graph is used to compare the size of one part to the whole or to another part.



10. What does Jack spend most of his time doing? sleeping

11. On what does Jack spend the least time? homework

12. Does he spend more time at school or at recreation? school

13. How many hours each day does Jack spend eating and doing other activities? 3 hours

2

MULTIPLICATION TABLE

1's	2's	3's	4's
1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36

TABLE OF AVERAGE PLANETARY DISTANCES

Planet	Average Distance From Sun	Average Distance From Earth
Mercury	·36 million	93 million
Venus	67 million	93 million
Earth	93 million	—
Mars	142 million	142 million
Jupiter	483 million	434 million
Saturn	886 million	886 million

All distances are rounded to nearest million miles.

15. How far from the sun is Saturn? 886 million miles

16. How far would you have to travel to get from the Earth to Mars? 142 million miles

17. Which would be shorter, a trip from the sun to Jupiter, or Jupiter to Saturn? Jupiter to Saturn

14. Which planet is closer to the Earth than it is to the sun? Jupiter

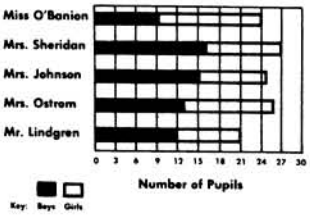
18. How much farther from the sun is Saturn than Mercury? 850 million miles

3

1. What is the average of: 36, 40, 54, 26?
First step: 36 + 40 + 54 + 26 = 156
Second step: 156 ÷ 4 = 39

2. What is the average of: 8, 10, 12, 14, 6, 10?
First step: 8 + 10 + 12 + 14 + 6 + 10 = 60
Second step: 60 ÷ 6 = 10

ENROLLMENT IN SIXTH GRADE CLASSES

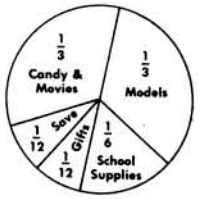


3. This is called a bar graph.

4. Which class has the largest number of boys? Mrs. Sheridan's

5. Which class has the most students? Mrs. Sheridan's class

SPENDING JOHN'S ALLOWANCE



6. The figure at the left is called a circle graph.

7. John's allowance is 60¢. How much does he spend on models? 20¢

8. Does John spend more money for candy and movies, or for school supplies? candy and movies

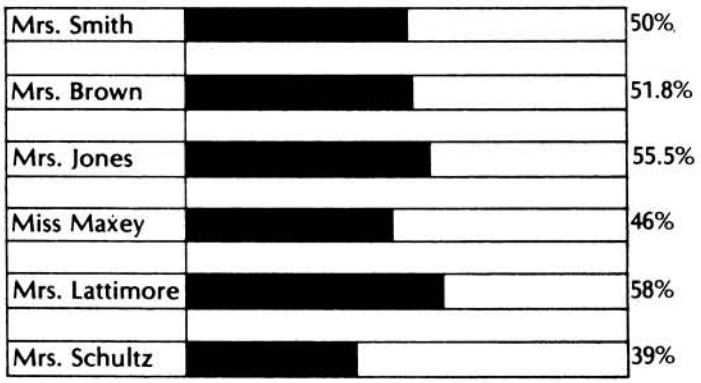
9. How much money does John save from his allowance? 5¢

4

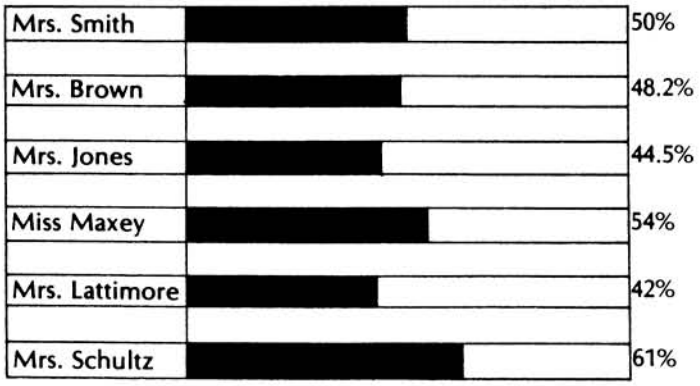
1. Introduce the following statistical information and have students use it to compute the percent of girls and boys in each class. Have them compare results to make sure the percents of girls and boys add up to 100 percent for each class; then have them graph the percentages on the appropriate chart.

Mrs. Smith's class	15 girls	50%
	15 boys	50%
Mrs. Brown's class	14 girls	51.8%
	13 boys	48.2%
Mrs. Jones' class	15 girls	55.5%
	12 boys	44.5%
Miss Maxey's class	12 girls	46%
	14 boys	54%
Mrs. Lattimore's class	18 girls	58%
	13 boys	42%
Mrs. Schultz's class	12 girls	39%
	19 boys	61%

PERCENT OF GIRLS



PERCENT OF BOYS



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